🔥 Laplace Transform (Unit-4 Mathematics-III) detailed and easy notes

⭐⭐⭐⭐⭐ **Very High Probability**

🔴 Most Important Topic of Unit-4

🔴 RGPV Favourite 7 Marks + 14 Marks Question

What Is Laplace Transform?

Laplace Transform is a mathematical technique used to convert a difficult differential equation into a simple algebraic equation.

Simple Meaning:

```
Differential Equation
        ↓
Laplace Transform
        ↓
Algebraic Equation
        ↓
Easy Solution
```

It converts a function from **time domain (t)** to **s-domain**.

Why Do We Need It?

Suppose we have:

$d^2y/dt^2+5dy/dt+6y=0$

This differential equation is difficult.

Laplace Transform converts it into a simple equation which is easier to solve.

Real-Life Example

Imagine a large Hindi paragraph.

You convert it into short notes.

Understanding becomes easier.

Similarly:

```
Complex Differential Equation
        ↓
Simple Algebraic Equation
```

This is the work of Laplace Transform.

Definition

The Laplace Transform of a function f(t) is defined as:

L{f(t)}=F(s)= Limit from 0 to ∞ ∫e^−st f(t)dt

Meaning of Symbols

L

Laplace Operator

f(t)

Function in time domain

F(s)

Function in s-domain

e

Exponential constant

s

Complex variable

dt

Small change in time

Basic Idea

Input:

```
f(t)
```

Apply Laplace Transform

Output:

```
F(s)
```

Example:

f(t)=1

After transform:

L{1}=1/s

Flowchart

```
Function f(t)
      |
      ▼
Apply Laplace
      |
      ▼
Get F(s)
      |
      ▼
Solve Easily
```

Standard Laplace Transforms

1. Constant Function

L{1}=1/s

---

2. t

$L\{t\}=1/s^2$

---

3. t^2

$L\{t^2\}=2!/s^3$

$=2/s^3$

---

4. e^{at}

$L\{e^{at}\}=1/s-a$

---

5. Sin Function

$L\{sinat\}=a/s^2+a^2$

---

6. Cos Function

$L\{cosat\}=s/s2+a2$

Most Important Formula Table

Function	Laplace Transform
1	$1/s$
t	$1/s^2$
t^2	$2/s^3$
e^{at}	$1/(s-a)$
sin at	$a/(s^2+a^2)$
cos at	$s/(s^2+a^2)$

🔥 Learn this table by heart.

Example

Find:

$L\{3t^2\}$

Using:

$L\{t^2\}=2/s^3$

Therefore:

$L\{3t^2\}=3\times2/s^3$

$=6/s^3$

Answer:

$=6/s^3$

Diagram

```
Time Domain
f(t)

    |

    ▼

Laplace Transform

    |

    ▼
```

```
s-Domain
F(s)
```

Advantages

✅ Simplifies differential equations

✅ Saves time

✅ Easy calculations

✅ Widely used in engineering

✅ Converts calculus into algebra

Disadvantages

❌ Formula memorization required

❌ Some transforms lengthy

❌ Inverse process needed

❌ Complex for beginners

❌ Requires transform tables

Applications

Electrical Engineering

Circuit Analysis

Mechanical Engineering

Vibration Analysis

Control Systems

System Modelling

Communication

Signal Processing

Civil Engineering

Structural Analysis

Frequently Asked RGPV Questions

7 Marks

1. Define Laplace Transform.
2. Explain Laplace Transform with examples.
3. Find Laplace Transform of standard functions.

---

14 Marks

1. Explain Laplace Transform and derive basic formulas.
2. Find Laplace Transform of various functions.
3. Explain applications of Laplace Transform.

Common Mistakes Students Make

❌ Forgetting transform formulas.

❌ Confusing sin and cos formulas.

❌ Wrong sign in eate^{at}eat.

❌ Missing factorial in tnt^ntn.

❌ Writing time-domain answer instead of s-domain.

Memory Trick

Remember:

$1 \to 1/s$

$t \to 1/s^2$

$t^2 \to 2/s^3$

Power increases:

s^1

s^2

s^3

Easy Rule:

```
Power of t
      ↓
Power of s increases by 1
```

2-Minute Revision

1. Laplace Transform converts time-domain to s-domain.
2. Used to solve differential equations.
3. Basic formula:

L{f(t)}= limit from 0 to ∞∫e^−st f(t)dt

4. L{1}=1/s
5. L{t}=1/s^2
6. L{e^at}=1/(s−a)
7. L{sinat}=a/(s^2+a^2)
8. L{cosat}=s/(s^2+a^2)
9. Used in circuits and control systems.
10. Most important topic of Unit-4.

Exam Focus

⭐⭐⭐⭐⭐ Very High Probability

Most Important Definition

Laplace Transform converts a function from time domain into s-domain.

Most Important Formula

L{f(t)}= limit from 0 to ∞∫e^−st f(t)dt

🔥 Properties of Laplace Transform (Unit-4 Mathematics-III)

⭐⭐⭐⭐⭐ **Very High Probability**

🔴 RGPV Favourite Theory Question

🔴 Frequently Asked in 7 Marks & 14 Marks

What Are Properties of Laplace Transform?

Properties are special rules that make Laplace Transform calculations easier.

Instead of solving every problem using the definition formula, we use these shortcut rules.

Think of them as:

```
Multiplication Table
for
Laplace Transform
```

Why Do We Need Them?

Without properties:

```
Every question
↓
Long Integration
```

```
↓
More Time
```

With properties:

```
Apply Formula
↓
Get Answer Quickly
```

---

1. Linearity Property

Statement

If

$L\{f(t)\}=F(s)$

and

$L\{g(t)\}=G(s)$

then

$L\{af(t)+bg(t)\}=aF(s)+bG(s)$

Simple Meaning

Laplace of a sum = Sum of Laplace

Example

Find:

$L\{3t+2\}$

Using:

$L\{t\}=1/s^2$

$L\{1\}=1/s$

Therefore

$L\{3t+2\}=3/s^2+2/s$

Memory Trick

```
Addition remains Addition
```

---

2. First Shifting Property

Formula

$L\{e^{at} f(t)\}=F(s-a)$

Simple Meaning

Whenever:

```
e^(at)
```

appears,

replace

```
s by (s-a)
```

Example

Find:

L{e^2tsin3t}

We know:

L{sin3t}= 3/s2+9

Replace:

s→(s−2)

Answer:

3/(s−2)^2+9

Memory Trick

```
e^(at)
↓
Shift s
```

---

3. Second Shifting Property

Formula

$L\{f(t-a)u(t-a)\}=e^{-as} F(s)$

Simple Meaning

Used when function starts after some delay.

Application

Signal Processing

Communication Systems

---

4. Differentiation Property

Formula

$L\{t\ f(t)\}=-\frac{d}{ds}[F(s)]$

Simple Meaning

Multiplication by t in time domain

becomes

Differentiation in s-domain.

Example

Find:

$L\{tsinat\}$

Since

$L\{sinat\}=a/s^2+a^2$

Differentiate w.r.t. s.

Memory Trick

```
t
↓
Differentiate
```

---

5. Division by t Property

Formula

$L\{f(t)/t\}=$ limit from s to $\infty \int F(u)\, du$

Use

Rarely asked.

Only theoretical importance.

---

6. Integration Property

Formula

L{limit from 0 to t ∫f(u) du}=F(s)/s

Simple Meaning

Integration in time domain

becomes

Division by s

Memory Trick

```
Integration
↓
Divide by s
```

---

7. Differentiation in Time Domain

Formula

L{df/dt}=sF(s)−f(0)

Most Important Property

⭐⭐⭐⭐⭐

Asked Every Year

Second Derivative

L{d^2f/dt^2}=s^2F(s)−sf(0)−f′(0)

Why Important?

Used in:

```
ODE by Laplace Transform
```

which is the most important Unit-4 numerical.

Example

Find:

L{dy/dt}

Answer:

sY(s)−y(0)

---

8. Initial Value Theorem

Formula

$f(0)=\lim_{s\to\infty} sF(s)$

Purpose

Find initial value quickly.

---

9. Final Value Theorem

Formula

$f(\infty)=\lim_{s\to 0} sF(s)$

Purpose

Find final steady-state value.

Most Important Properties for Exam

Property	**Importance**
Linearity	⭐⭐⭐⭐⭐
First Shifting	⭐⭐⭐⭐⭐
Differentiation	⭐⭐⭐⭐⭐
Integration	⭐⭐⭐⭐
Derivative Property	⭐⭐⭐⭐⭐
Initial Value Theorem	⭐⭐⭐
Final Value Theorem	⭐⭐⭐

Summary Table

Property	Formula
Linearity	L{af+bg}=aF+bG
First Shift	L{e^{at}f(t)}=F(s-a)
Second Shift	L{f(t-a)u(t-a)}=e^{-as}F(s)
Multiplication by t	L{tf(t)}=-dF/ds
Integration	L{∫fdt}=F(s)/s
First Derivative	sF(s)-f(0)
Second Derivative	s^2F(s)-sf(0)-f'(0)

Frequently Asked RGPV Questions

7 Marks

1. State and explain Linearity Property.
2. Explain First Shifting Property.
3. Explain Differentiation Property.
4. State Initial and Final Value Theorems.

---

14 Marks

1. Explain Properties of Laplace Transform with examples.
2. Prove First Shifting Property.
3. Explain Differentiation and Integration Properties.

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Most Repeated:

✅ Linearity Property

✅ First Shifting Property

✅ Derivative Property

✅ Integration Property

✅ Initial & Final Value Theorems

2-Minute Revision

```
Linearity
→ Addition remains Addition

e^(at)
→ Shift s

t.f(t)
→ Differentiate

Integral
→ Divide by s

First Derivative
→ sF(s)-f(0)

Second Derivative
→ s²F(s)-sf(0)-f'(0)

Initial Value
→ lim s→∞ sF(s)

Final Value
→ lim s→0 sF(s)
```

🔥 Laplace Transform of Periodic Functions

⭐⭐⭐⭐⭐ **Very High Probability**

🔴 RGPV Favourite 7-Mark Theory + Numerical Question

🔴 Frequently Asked in Mathematics-III Unit-4

What Is a Periodic Function?

A periodic function is a function that repeats itself after a fixed interval of time.

If:

$f(t+T)=f(t)$

then f(t) is called a periodic function.

Where:

- T = Period (time after which function repeats)

Real-Life Example

Clock

```
12 → 1 → 2 → ... → 12
```

```
Repeats every 12 hours.
```

Sine Wave

```
/\/\/\/\/\/\/\
```

Repeats continuously.

AC Current

Electricity in homes is periodic.

It repeats after a fixed time interval.

Why Do We Need Laplace Transform of Periodic Functions?

Many engineering signals are periodic:

- AC Voltage
- Sound Waves
- Radio Signals
- Vibrations

Direct Laplace Transform becomes difficult.

Therefore a special formula is used.

Definition

If

`f(t)`

has period

`T`

then

`f(t+T)=f(t)`

for all values of t.

Laplace Transform Formula for Periodic Function

🔥 Most Important Formula

L{f(t)}= 1/1−e ^−sT limit from 0 to T∫e^−st f(t)dt

Meaning of Symbols

T

Period of function

s

Laplace variable

e

Exponential constant

f(t)

Periodic function

Memory Trick

Remember:

```
Normal Laplace

↓

Periodic Formula

↓

Integral from 0 to T
```

↓

```
Divide by

(1-e^(-sT))
```

Flowchart

```
Check Function
       |
       ▼
Find Period T
       |
       ▼
Apply Periodic Formula
       |
       ▼
Integrate from 0 to T
       |
       ▼
Final Answer
```

Example 1

Given:

$f(t)=1$

for

$0<t<2$

and repeats after

$T=2$

Using Formula:

$L\{f(t)\}= 1/1-e^{-2s}$ limit from 0 to $2\int e^{-st}\,dt$

Integrating:

$=1/1-e^{-2s}[1-e-2s/s]$

Answer:

$= 1/s$

Graph of Periodic Function

```
1 ──────────┐      ──────────┐
        |          |
```

```
0──────────────────┴─────────────────────────────┴──────────→ t
      T=2           T=2
```

Repeats every 2 units.

Important Steps for Numerical Problems

Step 1

Find period

`T`

Step 2

Write periodic formula.

Step 3

Substitute function.

Step 4

Integrate from

```
0 → T
```

Step 5

Simplify answer.

Advantages

✅ Simplifies periodic signal analysis

✅ Useful in electrical engineering

✅ Useful in communication systems

✅ Easy formula available

✅ Frequently asked in exams

Disadvantages

❌ Integration required

❌ Long calculations

❌ Formula memorization needed

❌ Difficult for beginners

❌ Time consuming numericals

Applications

Electrical Engineering

AC circuits

Electronics

Periodic signals

Communication

Wave analysis

Mechanical Engineering

Vibration analysis

Control Systems

Repeating inputs

Difference Between Ordinary and Periodic Functions

Ordinary Function	Periodic Function
Does not repeat	Repeats after T
Normal Laplace	Special Formula
Single interval	Infinite repetition
Simpler	Slightly complex

Frequently Asked RGPV Questions

7 Marks

1. Define Periodic Function.
2. Derive Laplace Transform of Periodic Function.
3. State formula of Laplace Transform of Periodic Function.

---

14 Marks

1. Find Laplace Transform of a given Periodic Function.

2. Derive the formula for Laplace Transform of Periodic Functions.
3. Explain periodic functions with examples.

Most Important Definition

A function satisfying:

`f(t+T)=f(t)`

is called a periodic function.

Most Important Formula

`L{f(t)}=1/1−e^−sT limit from 0 to T∫e^−st f(t)dt`

🔥 Learn this formula exactly.

Most Expected 7-Mark Question

Define a periodic function and obtain its Laplace Transform.

Most Expected 14-Mark Question

Find the Laplace Transform of a periodic function using the standard formula.

2-Minute Revision

1. Periodic function repeats after time T.
2. Condition:

```
f(t+T)=f(t)
```

3. Common examples:
    - AC current
    - Sine wave
    - Clock motion
4. Use special Laplace formula.
5. Integral limits
6. 0 → T
7. Divide by:

```
1−e^−sT
```

```
Very important RGPV topic.
Frequently asked in theory and numericals.
Learn the formula by heart.
High-scoring 7-mark question.
```

🚀 **Exam Tip:** If you're short on time, memorize:

- Definition of periodic function
- Formula of Laplace Transform of periodic function
- One solved example

This alone is usually enough to score good marks on periodic-function questions in RGPV Mathematics-III.

🔥 Inverse Laplace Transform (ILT) – Different Methods

⭐⭐⭐⭐⭐ **Very High Probability**

🔴 RGPV Favourite 7-Mark + 14-Mark Topic

🔴 Every Year Questions Asked

What Is Inverse Laplace Transform?

Laplace Transform converts:

```
Time Domain
f(t)

↓

s-Domain
F(s)
```

Inverse Laplace Transform does the opposite.

```
s-Domain
F(s)

↓

Time Domain
f(t)
```

Why Do We Need It?

When solving differential equations:

Step 1

Apply Laplace Transform

Step 2

Solve algebraic equation

Step 3

Apply Inverse Laplace Transform

Step 4

Get final answer

Real-Life Example

Suppose:

```
English Sentence
↓
Hindi Translation
```

Laplace Transform.

Now:

```
Hindi
↓
English
```

Inverse Laplace Transform.

Definition

If

```
L{f(t)}= F(s)
```

then

```
L^−1{F(s)}= f(t)
```

Methods of Finding Inverse Laplace Transform

There are mainly:

1. Direct Formula Method

2. Partial Fraction Method

3. Convolution Method

4. Shifting Theorem Method

5. Division Method

Method 1: Direct Formula Method

Basic Idea

Compare with standard Laplace table.

Example

Find:

`L^−1{1/s}`

We know:

`L{1}= 1/s`

Therefore:

`L^−1{1/s}=1`

Important Direct Formulas

F(s)	**f(t)**
1/s	1
$1/s^2$	t
$n!/s^{n+1}$	t^n
1/(s-a)	e^{at}

F(s)	f(t)
$a/(s^2+a^2)$	sin at
$s/(s^2+a^2)$	cos at

🔥 Learn this table.

Method 2: Partial Fraction Method

⭐⭐⭐⭐⭐ Most Important

When Used?

When denominator has factors.

Example:

```
1/(s+1)(s+2)
```

Step 1

Assume:

```
1/(s+1)(s+2)
=A/s+1+B/s+2
```

Step 2

Find A and B.

Result:

```
A=1
B=−1
```

Step 3

Therefore:

```
=1/s+1−1/s+2
```

Step 4

Apply ILT.

```
e^−t−e^−2t
```

Answer:

```
e^−t−e^−2t
```

Memory Trick

```
Factor Denominator

↓

Break Fraction

↓

Apply Table
```

Method 3: Shifting Theorem Method

⭐⭐⭐⭐ Important

Formula

`L^−1{F(s−a)}=e^at f(t)`

Example

Find:

`L^−1{1/s−2}`

Answer:

`e^2t`

Method 4: Convolution Method

⭐⭐⭐⭐ Important

Formula

If

```
L^−1{F(s)}=f(t)
```

and

```
L^−1{G(s)}=g(t)
```

then

```
L^−1{F(s)G(s)}= limit from 0 to t ∫f(u)g(t−u)du
```

Use

Product of two transforms.

Example

```
1/s^2(s+1)
```

Often solved using convolution theorem.

Method 5: Long Division Method

⭐⭐⭐ Occasionally Asked

When Used?

When:

Degree Numerator

Degree Denominator

Example

`s^2+3s+2/s+1`

Use division first.

Then apply inverse transform.

Flowchart

```
Given F(s)
      |
      ▼
Check Form
      |
  ┌───────┼─────────┐
  ▼         ▼
Simple?  Factorized?
  |           |
  ▼           ▼
Direct   Partial Fraction
      ▼
Apply ILT
      ▼
Answer
```

Most Important Examples

Example 1

`L^−1{1/s^2}`

Answer:

`t`

Example 2

`L^−1{5/s^3}`

Answer:

`5/2t^2`

Example 3

`L^−1{3/s^2+9}`

Answer:

`sin3t`

Example 4

`L^−1{s/s^2+4}`

Answer:

`cos2t`

Advantages

✅ Solves differential equations

✅ Easy through tables

✅ Useful in engineering

✅ Multiple methods available

✅ High scoring topic

Applications

1. Differential Equations
2. Circuit Analysis
3. Control Systems
4. Signal Processing
5. Mechanical Vibrations

Frequently Asked RGPV Questions

7 Marks

1. Explain Inverse Laplace Transform.
2. Find ILT using Partial Fraction Method.
3. Explain Shifting Theorem.

---

14 Marks

1. Find ILT by different methods.
2. Explain Partial Fraction Method with examples.
3. Explain Convolution Method.

PYQ Analysis

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Repeated Questions:

✅ Partial Fraction Method

✅ Direct Formula Method

✅ Shifting Theorem

✅ Convolution Theorem

Most Important Formula

```
L^−1{F(s)}=f(t)
```

Most Expected 7-Mark Question

Find inverse Laplace transform using Partial Fraction Method.

Most Expected 14-Mark Question

Explain different methods of finding Inverse Laplace Transform with suitable examples.

2-Minute Revision

Methods

```
1. Direct Formula

2. Partial Fraction

3. Shifting Theorem

4. Convolution

5. Division Method
```

🔥 Convolution Theorem (Unit-4 Mathematics-III)

⭐⭐⭐⭐⭐ Very High Probability

🔴 RGPV Favourite 7-Mark & 14-Mark Question

🔴 Frequently Asked Theory + Numerical

What Is Convolution Theorem?

Convolution Theorem is used to find the **Inverse Laplace Transform of a product of two functions**.

Normally:

```
L{f(t)}=F(s)
L{g(t)}=G(s)
```

Then:

```
L^−1{F(s)G(s)}
```

can be found using Convolution Theorem.

Why Do We Need It?

Sometimes we get:

```
1/s^2(s+1)
```

or

```
1/s(s^2+a^2)
```

These are difficult to solve directly.

Convolution Theorem makes them easier.

Real-Life Example

Suppose:

```
Student A Work
+
Student B Work
=
Final Project
```

Similarly:

```
Function A
*
Function B
```

=

Convolution

Statement of Convolution Theorem

If

$L\{f(t)\}=F(s)$

and

$L\{g(t)\}=G(s)$

then

L^−1{F(s)G(s)}= limit from 0 to t ∫f(u)g(t−u) du

This integral is called **Convolution Integral**.

Most Important Formula

L^−1{F(s)G(s)}= limit from 0 to t ∫ f(u)g(t−u) du

Memory Trick

Remember:

```
Product in s-domain
```

↓

```
Convolution in time-domain
```

or

```
Multiply → Integrate
```

Basic Idea

Given:

```
F(s)G(s)
```

Find:

`f(t)`

and

`g(t)`

separately.

Then use:

`limit from 0 to t ∫f(u)g(t−u)du`

Flowchart

```
F(s)G(s)
     |
     ▼
Find f(t), g(t)
     |
     ▼
Apply Convolution Formula
     |
     ▼
Integrate
     |
     ▼
Answer
```

Example 1

Find:

```
L^−1{1/s(s+1)}
```

Step 1

Write:

```
F(s)=1/s
 G(s)=1/s+1
```

Step 2

Find inverse transforms.

```
L^−1{1/s}=1
 L^−1{1/s+1}=e^−t
```

Thus

```
f(t)=1
g(t)=e^−t
```

Step 3

Apply Convolution Formula

```
limit from 0 to t ∫1·e^−(t−u)du
```

Step 4

Take e−te^{-t}e−t common.

```
=e^−t lmit from 0 to t∫e^udu
```

Step 5

Integrate.

```
=e^−t[eu] limit from 0 to t
= e^{-t}(e^t−1)
```

Answer:

```
=1−e^−t
```

Diagram

```
F(s)
   |

  ×

   |

G(s)

   ↓

Convolution

   ↓

Integral

   ↓

Answer
```

Important Keywords for Exam

- Convolution Integral
- Product Theorem
- Inverse Laplace Transform
- Time Domain
- s-Domain
- Integration
- Product of Functions

Advantages

✅ Solves difficult inverse transforms

✅ Useful in differential equations

✅ Useful in signal processing

✅ Easy systematic approach

✅ Frequently asked in exams

Disadvantages

❌ Long calculations

❌ Integration required

❌ Time consuming

❌ Difficult for beginners

❌ Formula memorization needed

Applications

Electrical Engineering

Circuit Analysis

Communication Engineering

Signal Processing

Control Systems

System Response

Mechanical Engineering

Vibration Analysis

Mathematics

Differential Equations

Frequently Asked RGPV Questions

7 Marks

1. State Convolution Theorem.
2. Explain Convolution Integral.
3. Write formula of Convolution Theorem.

---

14 Marks

1. State and prove Convolution Theorem.
2. Find inverse Laplace Transform using Convolution Theorem.

3. Solve problems based on Convolution Theorem.

Most Important Definition

Convolution Theorem states that the inverse Laplace transform of the product of two transforms is equal to the convolution of their corresponding functions.

Most Expected 7-Mark Question

State and explain Convolution Theorem.

Most Expected 14-Mark Question

State and prove Convolution Theorem and solve a numerical example.

2-Minute Revision

Convolution Theorem

Product in s-domain
↓
Convolution in time-domain
↓
Integration
↓
Answer

Formula

L^−1{F(s)G(s)}= limit from 0 to t ∫f(u)g(t−u) du

Exam Shortcut

1/s → 1

1/(s+a) → e^(-at)

Use Formula

Integrate

Get Answer

🔥 Evaluation of Integrals by Laplace Transform

Mathematics-III (BT-401) Unit-4

⭐⭐⭐⭐ **High Probability**

🔴 Frequently Asked Theory + Numerical Question

What Is It?

Evaluation of Integrals by Laplace Transform means solving difficult definite integrals using Laplace Transform techniques instead of direct integration.

Sometimes integrals are very lengthy and difficult.

Laplace Transform makes them easier.

Why Do We Need It?

Suppose we have:

```
limit from 0 to ∞∫e^−atsin(bt)dt
```

Direct integration is lengthy.

Using Laplace Transform:

```
Transform Formula
       ↓
Direct Answer
```

Much easier.

Real-Life Example

Imagine calculating:

```
987 × 654
```

by hand.

Difficult.

Calculator makes it easy.

Similarly:

```
Integral
↓
Laplace Transform
↓
Easy Solution
```

Basic Idea

We know:

```
L{f(t)}= limit from 0 to ∞∫e^−stf(t)dt
```

This itself is an integral.

Therefore many definite integrals can be obtained directly from Laplace Transform tables.

Important Formula

If:

```
L{f(t)}=F(s)
```

then

```
limit from 0 to ∞ ∫e^−stf(t)dt=F(s)
```

Method to Solve Integrals

Step 1

Identify:

```
f(t)
```

Step 2

Find its Laplace Transform.

Step 3

Substitute value of sss.

Step 4

Get answer directly.

Example 1

Evaluate:

$\int e^{-2t} \sin 3t \, dt$

Step 1

We know:

`L{sin3t}=3/s^2+9`

Step 2

Compare with definition.

Here:

`s=2`

Step 3

Substitute:

`=3/2^2+9= \frac3{2^2+9}=22+93 =313= \frac3{13}=133`

Answer

`=3/13`

Example 2

Evaluate

```
limit from 0 to ∞∫e^−3tcos2t dt
```

Using:

```
L{cosat}=s/s^2+a^2
```

Substitute:

```
s=3, a=2
```

```
=3/9+4
 =3/13
```

Answer

```
=3/13
```

Most Important Standard Integrals

Integral 1

```
limit from 0 to ∞ ∫e^−stdt=1/s
```

Integral 2

```
limit from 0 to ∞ ∫e^−stsin(at)dt=a/s^2+a^2
```

Integral 3

```
limit from 0 to ∞ ∫e^−stcos(at)dt=s/s^2+a^2
```

Integral 4

```
limit from 0 to ∞  ∫te^−stdt=1/s2
```

Integral 5

```
limit from 0 to ∞ ∫t^2e−stdt= 2/s^3
```

Flowchart

```
Given Integral
        |
        ▼
Identify Function
        |
        ▼
Use Laplace Table
        |
        ▼
Substitute Value
        |
```

▼

`Answer`

Important Keywords for Exam

- Laplace Transform
- Definite Integral
- Infinite Limit
- Exponential Function
- Time Domain
- s-Domain

Advantages

✅ Simplifies integration

✅ Saves time

✅ Useful in engineering mathematics

✅ Easy calculation

✅ High scoring topic

Disadvantages

❌ Formula memorization needed

❌ Limited applications

❌ Requires transform knowledge

❌ Not useful for every integral

❌ Sometimes lengthy substitutions

Applications

1. Signal Processing

2. Control Systems
3. Circuit Analysis
4. Communication Engineering
5. Differential Equations

Frequently Asked RGPV Questions

7 Marks

1. Explain evaluation of integrals using Laplace Transform.
2. Evaluate standard integrals using Laplace Transform.

---

14 Marks

1. Evaluate definite integrals using Laplace Transform method.
2. Explain the procedure of evaluating integrals by Laplace Transform with examples.

Common Mistakes Students Make

❌ Forgetting standard Laplace formulas.

❌ Wrong substitution of sss.

❌ Confusing sin and cos transforms.

❌ Calculation mistakes.

❌ Not writing transform formula before solution.

Memory Trick

Remember:

```
sin(at)
↓
a/(s²+a²)

cos(at)
↓
s/(s²+a²)
```

↓
1/s

These three formulas solve most exam questions.

PYQ Analysis

⭐⭐⭐⭐ High Probability

Frequently Asked:

✅ Evaluate integral involving e−stsin(at)e^{-st}\sin(at)e−stsin(at)

✅ Evaluate integral involving e−stcos(at)e^{-st}\cos(at)e−stcos(at)

✅ Explain method of evaluating integrals using Laplace Transform

---

Most Important Formula

```
L{f(t)}= limit from 0 to ∞∫e^−stf(t)dt
```

Most Expected 7-Mark Question

Evaluate a definite integral using Laplace Transform.

Most Expected 14-Mark Question

Explain evaluation of integrals by Laplace Transform and solve suitable examples.

2-Minute Revision

1. Laplace Transform itself is an integral.
2. Difficult integrals can be solved using transform tables.
3. Identify function f(t).
4. Find Laplace Transform.
5. Substitute value of sss.
6. Get answer directly.
7. Learn transforms of:
    - 1

    - sinat
    - cosat

8. Frequently asked in RGPV.
9. Easy scoring topic.
10. Usually appears as a short or medium numerical question.

🔥 **Exam Shortcut:** Learn only these three formulas:

```
L{1}=1/sL
L{sinat}=a/s^2+a^2
L{cosat}=s/s^2+a^2
```

and you can solve most Laplace-transform integral questions asked in RGPV exams.

🔥 Solving ODEs by Laplace Transform Method

Mathematics-III (BT-401) Unit-4

⭐⭐⭐⭐⭐ **VERY HIGH PROBABILITY**

🔴 Most Important Numerical Topic of Unit-4

🔴 Almost Every Year Asked in RGPV

What Is It?

Laplace Transform Method is used to solve Ordinary Differential Equations (ODEs).

Instead of solving the differential equation directly, we:

```
Differential Equation
        ↓
Laplace Transform
        ↓
Algebraic Equation
        ↓
Solve
        ↓
Inverse Laplace Transform
        ↓
Final Answer
```

Why Do We Need It?

Suppose:

```
dy/dt+2y=3
```

Direct solution may be lengthy.

Laplace Transform converts it into a simple algebraic equation.

Real-Life Example

Imagine:

```
Big Complicated Paragraph
        ↓
Short Notes
        ↓
Easy Understanding
```

Laplace Transform does the same with differential equations.

Most Important Formulas

First Derivative

```
L{dy/dt}=sY(s)−y(0)
```

Second Derivative

```
L{d^2y/dt^2}=s^2Y(s)−sy(0)−y′(0)
```

🔥 These two formulas are used in almost every numerical.

General Procedure

Step 1

Write ODE.

Step 2

Take Laplace Transform of both sides.

Step 3

Use derivative formulas.

Step 4

Substitute initial conditions.

Step 5

Find Y(s)

Step 6

Apply Inverse Laplace Transform.

Step 7

Get final answer y(t).

Solved Example (Most Important)

Solve:

```
dy/dt+y=0
```

Given:

```
y(0)=1
```

Step 1

Take Laplace Transform.

```
L{dy/dt}+L{y}=0
```

Step 2

Apply formula.

```
sY(s)−y(0)+Y(s)=0
```

Step 3

Substitute:

```
y(0)=1
sY(s)−1+Y(s)=0
```

Step 4

Collect terms.

```
Y(s)(s+1)=1
```

Step 5

Find Y(s)Y(s)Y(s).

```
Y(s)=1/s+1
```

Step 6

Take Inverse Laplace Transform.

We know:

```
L^−1{1/s+1}=e^−t
```

Final Answer

```
y=e^−t
```

Flowchart

```
Given ODE
       |
       ▼
Take Laplace
       |
       ▼
Use Derivative Formula
       |
       ▼
Apply Initial Conditions
       |
       ▼
Find Y(s)
       |
       ▼
Take Inverse Laplace
       |
       ▼
Final Answer
```

Important Formula Table

ODE Term	Laplace Transform
y	$Y(s)$
dy/dt	$sY(s)-y(0)$
d^2y/dt^2	$s^2Y(s)-sy(0)-y'(0)$
d^3y/dt^3	$s^3Y(s)-s^2y(0)-sy'(0)-y''(0)$

🔥 Learn this table.

Example 2

Solve:

```
d^2y/dt^2+y=0
```

Given:

```
y(0)=0
y´(0)=1
```

Taking Laplace

```
s^2Y(s)−1+Y(s)=0
```

```
Y(s)=1/s^2+1
```

Taking ILT:

```
y=sint
```

Advantages

✅ Easy for differential equations

✅ Uses initial conditions directly

✅ Converts calculus into algebra

✅ High accuracy

✅ Widely used in engineering

Applications

Electrical Circuits

Circuit Analysis

Mechanical Engineering

Vibration Problems

Control Systems

System Response

Communication

Signal Analysis

Mathematics

ODE Solutions

Common Mistakes Students Make

❌ Forgetting initial conditions.

❌ Wrong derivative formula.

❌ Mistakes in inverse Laplace.

❌ Missing negative sign.

❌ Wrong partial fractions.

Frequently Asked RGPV Questions

7 Marks

1. Explain solution of ODE using Laplace Transform.
2. State Laplace Transform of derivatives.

---

14 Marks

1. Solve first-order differential equation using Laplace Transform.
2. Solve second-order differential equation using Laplace Transform.
3. Explain the complete procedure of solving ODE by Laplace Transform.

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Repeated Questions:

✅ Solve ODE using Laplace Transform.

✅ Laplace Transform of Derivatives.

✅ Initial Value Problems.

✅ Second Order ODE using Laplace.

Most Important Formula

`L{dy/dt}=sY(s)−y(0)`

Most Expected 7-Mark Question

Explain the method of solving ODE using Laplace Transform.

Most Expected 14-Mark Question

Solve a second-order differential equation using Laplace Transform method.

2-Minute Revision

```
Take Laplace

↓

Use Derivative Formula

↓

Apply Initial Conditions

↓

Find Y(s)

↓

Take Inverse Laplace

↓

Answer
```

Learn These Two Formulas

```
L{y′}=sY(s)−y(0)
L{y″}=s^2Y(s)−sy(0)−y′(0)
```

🔥 Fourier Transform (Unit-4 Mathematics-III)

⭐⭐⭐⭐ **High Probability**

🟡 Usually Asked as Theory Question

🟡 Easy 7-Mark Topic

🟡 Important for Viva and Short Notes

What Is Fourier Transform?

Fourier Transform is a mathematical tool used to convert a function from the **time domain** into the **frequency domain**.

Simple Meaning:

```
Signal in Time
      ↓
Fourier Transform
      ↓
Signal in Frequency
```

It tells us what frequencies are present in a signal.

Why Do We Need It?

Suppose a music signal contains:

- Bass Sound

- Drum Sound
- Guitar Sound

All mixed together.

Fourier Transform separates these frequencies.

Real-Life Example

Imagine white light.

White light contains:

```
Red
Orange
Yellow
Green
Blue
Indigo
Violet
```

Prism separates them.

Similarly:

```
Complex Signal
       ↓
Fourier Transform
       ↓
Individual Frequencies
```

Basic Idea

A complicated signal can be represented as a combination of:

```
Sine Waves

+

Cosine Waves
```

Fourier Transform finds those sine and cosine components.

Definition

The Fourier Transform of a function f(x) is:

```
F(ω)= limit from −∞ to ∞ ∫f(x)e^−iωxdx
```

Where:

- f(x) = original function
- F(ω)= transformed function
- ω = angular frequency
- i= sqrt{-1}

Inverse Fourier Transform

Used to return back to original function.

```
f(x)=1/2π∫−∞∞F(ω)e^iωxdω
```

Meaning of Terms

Frequency

How fast a signal repeats.

Time Domain

Signal represented with time.

Example:

```
Voltage vs Time
```

Frequency Domain

Signal represented with frequency.

Example:

```
Amplitude vs Frequency
```

Diagram

```
Time Domain
f(t)

     |
```

```
▼

Fourier Transform

    |

▼

Frequency Domain
F(ω)
```

Fourier Transform vs Laplace Transform

Fourier Transform	Laplace Transform
Time → Frequency	Time → s-domain
Used for signal analysis	Used for ODE solving
Frequency information	Differential equation solution
Communication systems	Control systems
Uses ω	Uses s

Standard Fourier Transforms

Constant Function

```
F{1}=2πδ(ω)
```

Exponential Function

```
F{e^−ax}=1/a+iω
```

Cos Function

$$F\{\cos ax\} = \pi[\delta(\omega - a) + \delta(\omega + a)]$$

Sin Function

$$F\{\sin ax\} = \pi / i[\delta(\omega - a) - \delta(\omega + a)]$$

(Usually not asked in detail in RGPV)

Properties of Fourier Transform

1. Linearity

$$F\{af + bg\} = aF + bG$$

---

2. Shifting Property

Shift in time causes phase shift.

---

3. Scaling Property

Stretching signal changes frequency.

---

4. Differentiation Property

Differentiation in time domain becomes multiplication in frequency domain.

Advantages

✅ Signal analysis

✅ Frequency identification

✅ Communication systems

✅ Image processing

✅ Noise filtering

Disadvantages

❌ Complex calculations

❌ Difficult mathematics

❌ Infinite limits

❌ Not suitable for beginners

❌ Requires transform tables

Applications

Communication Engineering

Signal transmission

Image Processing

Image enhancement

Audio Processing

Music analysis

Electronics

Filter design

Wireless Communication

Spectrum analysis

Medical Engineering

MRI and CT scan processing

Frequently Asked RGPV Questions

7 Marks

1. Define Fourier Transform.
2. Explain Fourier Transform with applications.
3. Difference between Laplace and Fourier Transform.

---

14 Marks

1. Explain Fourier Transform and its properties.
2. Derive Fourier Transform.
3. Explain applications of Fourier Transform in engineering.

Common Mistakes Students Make

❌ Confusing Fourier and Laplace Transform.

❌ Forgetting frequency domain concept.

❌ Not writing applications.

❌ Missing definition.

❌ Writing Laplace formula instead of Fourier formula.

Memory Trick

Remember:

```
Laplace
=
Differential Equations

Fourier
=
Frequency Analysis
```

OR

```
L → Logic for ODE

F → Frequency
```

PYQ Analysis

⭐⭐⭐⭐ High Probability

Repeated Questions:

✅ Definition of Fourier Transform

✅ Applications of Fourier Transform

✅ Difference between Fourier and Laplace Transform

✅ Properties of Fourier Transform

Most Important Definition

Fourier Transform is a mathematical technique used to convert a signal from time domain to frequency domain.

Most Important Formula

```
F(ω)= −∞∞ ∫f(x)e^−iωxdx
```

Most Expected 7-Mark Question

Define Fourier Transform and explain its applications.

Most Expected 14-Mark Question

Explain Fourier Transform, its properties, and applications in engineering.

2-Minute Revision

Fourier Transform

```
Time Domain
      ↓
Frequency Domain
```

Used For

✅ Signal Analysis

✅ Communication

✅ Image Processing

✅ Audio Processing

Formula

```
F(ω)= limit from −∞ to ∞∫f(x)e^−iωxdx
```

Difference

```
Laplace
→ ODE Solution

Fourier
→ Frequency Analysis
```