

UNIT -03 M3 NOTES WITH EASY EXPLANATION AND DIAGRAMS

Taylor's Series Method (Unit-3 Mathematics-III)

● RGPV Importance: VERY HIGH

★★★★★ Frequently Asked Theory + Numerical

What Is It?

Taylor's Series Method is a numerical method used to find the approximate solution of a differential equation.

When solving a differential equation exactly becomes difficult, Taylor's Series helps us find an approximate value of y .

It uses derivatives (y', y'', y''' , etc.) to predict future values.

Why Do We Need It?

Suppose we have:

$$dy/dx = x + y$$

and

$$y(0) = 1$$

Finding the exact solution may be difficult.

Taylor's Method helps us find approximate values like:

$y(0.1), y(0.2), y(0.3)$

easily.

Real-Life Example

Imagine you are driving a car.

You know:

- Current position
- Current speed
- Current acceleration

Using these values, you can predict where the car will be after a few seconds.

Taylor Method works similarly.

Current value + rate of change + higher changes = future value.

Basic Idea

Taylor Series expands a function around a known point.

General Taylor Expansion:

$$y(x+h) = y(x) + hy' + \frac{h^2}{2!}y'' + \frac{h^3}{3!}y''' + \dots$$

This formula gives the next value of y .

Important Formula

$$y(x+h)=y+hy'+h^2/2!y''+h^3/3!y''' +h^4/4!y''''+\dots$$

Where:

- y = present value
 - h = step size
 - y' = first derivative
 - y'' = second derivative
 - y''' = third derivative
-

Step-by-Step Explanation

Suppose:

$$dy/dx=x+y$$

and

$$y(0)=1$$

Find:

$$y(0.1)$$

Step 1

Given:

$$y'=x+y$$

At:

$$x=0,y=1$$

$$y'=0+1=1$$

Step 2

Find second derivative.

Differentiate:

$$y' = x + y$$

$$y'' = 1 + y'$$

Substitute:

$$y'' = 1 + 1 = 2$$

Step 3

Find third derivative.

$$y''' = y''$$

$$y''' = 2$$

Step 4

Use Taylor Formula.

$$y(0.1) = y + hy' + \frac{h^2}{2}y'' + \frac{h^3}{6}y'''$$

Substitute:

$$= 1 + 0.1(1) + \frac{(0.1)^2}{2}(2) + \frac{(0.1)^3}{6}(2)$$

$$= 1 + 0.1 + 0.01 + 0.000333$$

$$y(0.1) = 1.11033$$

Diagram / Flowchart

Given Differential Equation



Find y'

|



Find y''

|



Find y'''

|



Substitute in Taylor Formula

|



Obtain Approximate Value

Important Keywords for Exam

- Numerical Method
- Approximate Solution
- Differential Equation
- Taylor Expansion
- Higher Derivatives
- Step Size
- Initial Condition

Advantages

- ✓ High accuracy
- ✓ Simple concept
- ✓ Good for small intervals
- ✓ Gives exact trend

✓ Basis of many numerical methods

Disadvantages

- ✗ Need higher derivatives
 - ✗ Long calculations
 - ✗ Difficult for complex equations
 - ✗ Time-consuming
 - ✗ Not suitable for large intervals
-

Applications

1. Engineering calculations
 2. Physics simulations
 3. Heat transfer
 4. Population growth
 5. Electrical systems
-

Frequently Asked RGPV Questions

1. Explain Taylor's Series Method.
 2. Derive Taylor Series Formula.
 3. Solve differential equation using Taylor Method.
 4. Find $y(0.1)$ using Taylor Series.
 5. Compare Taylor and Euler Method.
-

Common Mistakes Students Make

✗ Forgetting to calculate higher derivatives.

✗ Wrong differentiation.

✗ Substituting wrong values of x and y .

✗ Ignoring factorial terms.

✗ Using wrong step size h .

2-Minute Revision

1. Taylor Method solves ODE numerically.
 2. Uses derivatives.
 3. Gives approximate solution.
 4. Requires y', y'', y''' .
 5. Based on Taylor Expansion.
 6. Highly accurate.
 7. Works for small intervals.
 8. Formula starts with $y + hy'$
 9. Factorials are important.
 10. Frequently asked in RGPV.
 11. Theory + Numerical both important.
 12. Base method for many advanced techniques.
-

Memory Trick

"Taylor = Future Prediction"

Think:

Present Value + Change + More Change + More Change

Present
+
1st Change
+
2nd Change
+
3rd Change
=
Future Value

Remember:

Taylor predicts future values using present derivatives.

Exam Focus

★ PYQ Frequency: High

★★★★★ Very High Probability

★ Most Important Formula:

$$y(x+h) = y + hy' + \frac{h^2}{2!}y'' + \frac{h^3}{3!}y''' + \dots$$

★ Most Expected 7-Mark Question:

"Explain Taylor's Series Method with suitable example."

★ Most Expected 14-Mark Question:

"Solve a differential equation using Taylor's Series Method and explain the procedure."

Euler Method (Unit-3 Mathematics-III)

● **RGPV Importance: VERY HIGH**

★★★★★ Repeated Numerical + Theory Question

What Is It?

Euler Method is the simplest numerical method used to solve Ordinary Differential Equations (ODEs).

When exact solution is difficult, Euler Method helps us find approximate values of y .

It uses the slope of the curve to move step-by-step.

Why Do We Need It?

Suppose we have:

$$dy/dx = x + y$$

and

$$y(0) = 1$$

We want $y(0.1)$

Instead of solving the whole equation, Euler Method gives an approximate answer quickly.

Real-Life Example

Imagine you are walking on a hill.

You know:

- Current position
- Direction of slope

Using the slope, you take a small step and estimate where you'll reach next.

Euler Method works exactly like this.

Basic Idea

Slope means:

$$dy/dx$$

Euler says:

Future Value = Present Value + Change

Formula:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Where:

- y_n = current value
 - h = step size
 - $f(x, y)$ = given differential equation
-

Step-by-Step Procedure

Given:

$$dy/dx = x + y$$

$$y(0) = 1$$

Find:

$$y(0.1)$$

Step 1

Current values:

$$x_0=0$$

$$y_0=1$$

Step size:

$$h=0.1$$

Step 2

Find slope.

$$f(x,y)=x+y$$

$$f(0,1)=1$$

Step 3

Apply Euler Formula.

$$y_1=y_0+hf(x_0,y_0)$$

$$y_1=1+0.1(1)$$

Answer:

$$y(0.1)=1.1$$

Diagram

(x_0, y_0)



|

| slope = dy/dx

|



(x_1, y_1)

Move using slope

Important Keywords

- Numerical Method
 - Approximate Solution
 - Step Size
 - Slope
 - Differential Equation
 - Initial Value Problem
-

Advantages

- ✓ Very simple
 - ✓ Easy calculations
 - ✓ Quick method
 - ✓ Good for beginners
 - ✓ Frequently asked in exams
-

Disadvantages

- ✗ Less accurate
 - ✗ Error increases rapidly
 - ✗ Not suitable for large step sizes
 - ✗ Poor accuracy for curved functions
 - ✗ Approximation only
-

Applications

1. Population growth
 2. Heat transfer
 3. Electrical circuits
 4. Fluid flow
 5. Engineering simulations
-

Frequently Asked RGPV Questions

1. Explain Euler Method.
 2. Derive Euler Formula.
 3. Solve ODE using Euler Method.
 4. Advantages and disadvantages.
 5. Compare Euler and Modified Euler.
-

Memory Trick

"Euler = Slope Method"

Remember:

Future Value
=
Present Value
+
(Slope \times Step Size)

2-Minute Revision

1. Simplest numerical method.
2. Uses slope.
3. Solves ODE.
4. Approximate solution.
5. Formula:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

6. Easy but less accurate.
 7. Uses step size h .
 8. Frequently asked in RGPV.
-

Modified Euler Method

 **RGPV Importance: VERY HIGH**

 Frequently Asked Theory + Numerical

What Is It?

Modified Euler Method is an improved version of Euler Method.

Euler uses only one slope.

Modified Euler uses **two slopes** and takes their average.

Therefore, it gives better accuracy.

Why Do We Need It?

Euler Method may give errors.

Modified Euler reduces those errors.

It provides a value closer to the exact solution.

Real-Life Example

Suppose you are driving.

Euler Method:

Uses only current speed.

Modified Euler:

Uses current speed and future speed and then averages them.

Hence prediction becomes more accurate.

Basic Idea

Step 1:

Predict value using Euler.

Step 2:

Find new slope.

Step 3:

Average both slopes.

Step 4:

Correct answer.

Formula

Predictor

$$y_p = y_n + hf(x_n, y_n)$$

Corrector

$$y_{n+1} = y_n + h/2[f(x_n, y_n) + f(x_{n+1}, y_p)]$$

Step-by-Step Procedure

Given:

$$dy/dx = x + y$$

$$y(0) = 1$$

$$h = 0.1$$

Step 1 (Predictor)

$$y_p = 1 + 0.1(1)$$

$$y_p=1.1$$

Step 2

New point:

$$x_1=0.1$$

$$y_p=1.1$$

New slope:

$$f(0.1, 1.1)=1.2$$

Step 3 (Corrector)

$$y_1=1+0.1/2(1+1.2)$$

$$=1.11$$

Answer:

$$y(0.1)=1.11$$

Diagram

Initial Point

|



Predict (Euler)

|



Find New Slope

|



Average Slopes

|



Correct Answer

Difference Between Euler and Modified Euler

Euler	Modified Euler
Uses one slope	Uses two slopes
Less accurate	More accurate
Simple	Slightly lengthy
Higher error	Lower error
Basic method	Improved method

Advantages

- ✓ Better accuracy
- ✓ Less error
- ✓ Easy implementation
- ✓ Improved Euler
- ✓ Widely used

Disadvantages

- ✗ More calculations
- ✗ Slower than Euler

✗ Still approximate

✗ Not exact

✗ Repeated iterations needed

Applications

1. Engineering problems
 2. Heat transfer
 3. Fluid mechanics
 4. Control systems
 5. Mathematical modelling
-

Frequently Asked RGPV Questions

1. Explain Modified Euler Method.
 2. Derive Modified Euler Formula.
 3. Solve ODE using Modified Euler Method.
 4. Compare Euler and Modified Euler.
 5. Explain Predictor-Corrector concept.
-

Memory Trick

"Modified Euler = Predict + Correct"

Predict

↓

Check

↓

Correct



Final Answer

Exam Focus

Euler Method

★★★★★ Very High Probability

Most Expected 7-Mark Question:

Explain Euler Method with numerical example.

Most Expected 14-Mark Question:

Solve differential equation using Euler Method and discuss merits and demerits.

Modified Euler Method

★★★★★ Very High Probability

Most Expected 7-Mark Question:

Explain Modified Euler Method.


Most Expected 14-Mark Question:

Solve differential equation using Modified Euler Predictor-Corrector Method

Runge-Kutta Method of Fourth Order (RK-4 Method)

MOST IMPORTANT TOPIC OF UNIT-3

 Very High Probability

 RGPV Favourite Numerical

 Guaranteed 7-Mark / 14-Mark Question

What Is It?

Runge-Kutta Fourth Order Method (RK-4) is a numerical method used to solve Ordinary Differential Equations (ODEs).

It is an improved version of Euler Method.

Instead of using only one slope, RK-4 uses **four slopes** and then takes their weighted average.

Therefore it gives very accurate results.

Why Do We Need RK-4?

Euler Method:

Uses 1 slope

Less Accuracy

Modified Euler:

Uses 2 slopes

Better Accuracy

RK-4:

Uses 4 slopes

Very High Accuracy

Hence RK-4 is preferred in engineering calculations.

Real-Life Example

Suppose you want to predict tomorrow's weather.

Euler Method

Looks only at today's weather.

RK-4 Method

Looks at:

- Morning weather
- Noon weather
- Evening weather
- Night weather

Then combines all information.

Obviously prediction becomes better.

Basic Idea

RK-4 calculates:

k_1

k_2

k_3

k_4

These are four different slopes.

Then combines them.

Important Formula

For:

$$dy/dx=f(x,y)$$

First Slope

Second Slope

$$k_2=hf(x_n+h/2,y_n+k_1/2)$$

Third Slope

$$k_3=hf(x_n+h/2,y_n+k_2/2)$$

Fourth Slope

$$k_4=hf(x_n+h,y_n+k_3)$$

Final Formula

$$y_{n+1}=y_n+(k_1+2k_2+2k_3+k_4)/6$$

Memory Trick

Remember:

1 : 2 : 2 : 1

Means:

$k_1 + 2k_2 + 2k_3 + k_4$

6

This pattern is always used.

Step-by-Step Numerical

Given:

$dy/dx = x + y$

$y(0) = 1$

$h = 0.1$

Find:

$y(0.1)$

Step 1

$k_1 = 0.1(0 + 1)$

$k_1 = 0.1$

Step 2

$$k_2 = 0.1[0.05 + (1 + 0.1/2)]$$

$$k_2 = 0.11$$

Step 3

$$k_3 = 0.1[0.05 + (1 + 0.11/2)]$$

$$k_3 = 0.1105$$

Step 4

$$k_4 = 0.1[0.1 + (1 + 0.1105)]$$

$$k_4 = 0.12105$$

Step 5

Apply RK Formula

$$y_1 = 1 + (0.1 + 2(0.11) + 2(0.1105) + 0.12105)/6$$

$$y_1 = 1.11034$$

Answer:

$$y(0.1) = 1.11034$$

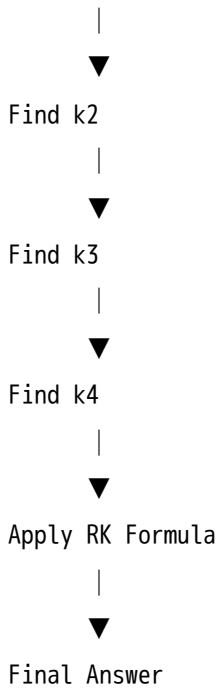
Flowchart

Given ODE

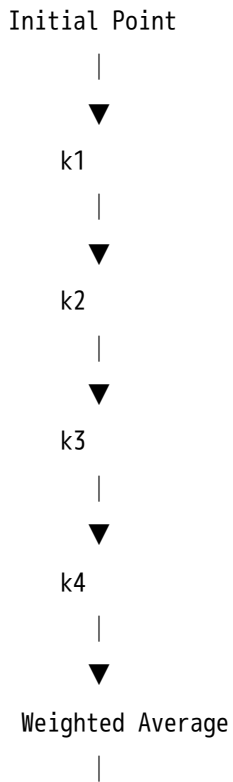
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Find k_1



Diagram





Final Value

Advantages

- ✓ Very accurate
 - ✓ Less error
 - ✓ Widely used
 - ✓ No higher derivatives needed
 - ✓ Suitable for engineering problems
-

Disadvantages

- ✗ More calculations
 - ✗ Time consuming manually
 - ✗ Complex for beginners
 - ✗ Large problems require computers
 - ✗ Longer numerical steps
-

Applications

1. Electrical Engineering
2. Mechanical Engineering
3. Heat Transfer
4. Population Growth Models

5. Fluid Mechanics
 6. Control Systems
 7. Aerospace Engineering
-

Frequently Asked RGPV Questions

7 Marks

1. Explain RK-4 Method.
 2. Advantages of RK-4.
 3. Difference between Euler and RK-4.
-

14 Marks

1. Derive RK-4 Formula.
 2. Solve ODE using RK-4 Method.
 3. Compare Euler, Modified Euler and RK-4.
-

Difference Between Euler and RK-4

Euler	RK-4
Uses 1 slope	Uses 4 slopes
Less accurate	Very accurate
Easy	Slightly lengthy
High error	Low error
Basic Method	Advanced Method

2-Minute Revision

1. RK-4 solves ODE numerically.

2. Uses four slopes.
 3. Slopes are k_1, k_2, k_3, k_4 .
 4. Most accurate method in Unit-3.
 5. Uses weighted average.
 6. Pattern = 1 : 2 : 2 : 1.
 7. No higher derivatives needed.
 8. Better than Euler.
 9. Better than Modified Euler.
 10. Frequently asked in RGPV.
 11. Numerical almost every year.
 12. Must prepare for exam.
-

RK-4 for Second Order Differential Equations

Suppose:

$$d^2y/dx^2=f(x,y,y')$$

Convert into two first-order equations.

Let

$$y'=z$$

Then

$$dy/dx=z$$

and

$$dz/dx=f(x,y,z)$$

Now apply RK-4 separately on:

$$dy/dx = z$$

$$dz/dx = f(x, y, z)$$

This is the standard procedure used in RGPV.

Exam Focus

★★★★★ Very High Probability

Most Important Formula

$$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

Most Important Diagram

RK-4 Flowchart

Most Expected 7-Mark Question

Explain Runge-Kutta Fourth Order Method.

Milne's Predictor-Corrector Method

🔥 RGPV Mathematics-III Unit-3 Important Topic

★★★★★ Very High Probability

● Frequently Asked in 7 Marks & 14 Marks

What Is It?

Milne's Method is a numerical method used to solve Ordinary Differential Equations (ODEs).

It belongs to the **Predictor-Corrector Family**.

This means:

Step 1 → Predict Answer

Step 2 → Correct Answer

First we estimate the value.

Then we improve that estimate.

Why Do We Need It?

Suppose Euler Method gives:

$$y=1.52$$

Actual answer is:

$$y=1.56$$

Milne Method improves the answer.

Therefore accuracy becomes better.

Real-Life Example

Suppose you predict tomorrow's temperature.

Prediction:

40°C

Then new weather information arrives.

You revise prediction:

38°C

This is Predictor-Corrector concept.

Basic Idea

Milne Method needs:

4 Previous Points

For example:

(x_0, y_0)

(x_1, y_1)

(x_2, y_2)

(x_3, y_3)

Using these values:

Predict y_4

Then Correct y_4

Flowchart

Known Values

|



Predict Next Value

|



Find Slope

|



Correct Value

|



Final Answer

Predictor Formula

The predicted value is:

$$y_{n+1}(p) = y_{n-3} + 4h/3(2f_{n-2} - f_{n-1} + 2f_n)$$

Corrector Formula

$$y_{n+1}(c) = y_{n-1} + h/3(f_{n-1} + 4f_n + f_{n+1})$$

Memory Trick

Milne

Predict First

Correct Later

Remember:

P → Prediction

C → Correction

Step-by-Step Procedure

Step 1

Find:

$$f=dy/dx$$

for all known points.

Step 2

Apply Predictor Formula.

Find:

$$y_{n+1}(p)$$

Step 3

Calculate:

f_{n+1}

using predicted value.

Step 4

Apply Corrector Formula.

Get final answer.

Diagram

x_0 ---- x_1 ---- x_2 ---- x_3 ---- x_4

Known Known Known Known

|



Predict

|



Correct

|



Final

Advantages

- ✓ High accuracy
 - ✓ Better than Euler
 - ✓ Less error
 - ✓ Useful for large calculations
 - ✓ Exam favourite topic
-

Disadvantages

- ✗ Requires previous values
 - ✗ Lengthy calculations
 - ✗ Difficult manually
 - ✗ Initial values needed
 - ✗ More formulas
-

Applications

1. Engineering calculations
 2. Population growth
 3. Heat transfer
 4. Fluid mechanics
 5. Scientific simulations
-

Frequently Asked RGPV Questions

7 Marks

1. Explain Milne Predictor-Corrector Method.
2. Write Predictor Formula.
3. Write Corrector Formula.

14 Marks

1. Solve ODE using Milne Method.
 2. Explain Predictor-Corrector concept with example.
-

Adams Predictor-Corrector Method

 **Most Important Theory Topic**

 Very High Probability

What Is It?

Adams Method is another Predictor-Corrector Method.

Like Milne:

Predict

↓

Correct

↓

Final Answer

But formulas are different.

Why Do We Need It?

It gives:

More Accuracy

Less Error

Better Stability

than many simple methods.

Real-Life Example

Suppose a cricket coach predicts score:

250 Runs

After seeing current match condition:

245 Runs

Prediction is corrected.

This is Adams Method.

Basic Idea

Uses:

Past Values

+

Past Slopes

to predict future value.

Adams Predictor Formula

$$y_{n+1} = y_n + h/24(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

Adams Corrector Formula

$$y_{n+1} = y_n + h/24(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2})$$

Easy Memory Trick

Adams Formula Coefficients

Predictor:

55 59 37 9

Corrector:

9 19 5 1

Remember:

55-59-37-9

9-19-5-1

These are frequently asked in exams.

Flowchart

Known Values

|



Predict Next Value

|



Find New Slope

|



Correct Value

|



Final Answer

Difference Between Milne and Adams

Milne	Adams
Uses Milne Formula	Uses Adams Formula
Simpler	More accurate
Less stable	More stable
Common in textbooks	Common in practical use
Predictor-Corrector	Predictor-Corrector

Advantages

- ✓ High accuracy
- ✓ Less truncation error
- ✓ Stable method
- ✓ Suitable for computers
- ✓ Better prediction

Disadvantages

- ✗ Formula lengthy
- ✗ Initial values required
- ✗ More calculations
- ✗ Difficult manually
- ✗ Not suitable without starting values

Frequently Asked RGPV Questions

7 Marks

1. Explain Adams Predictor-Corrector Method.
2. Difference between Milne and Adams.

14 Marks

1. Solve ODE using Adams Method.
 2. Explain Predictor-Corrector Methods.
-

PYQ Analysis

MUST STUDY

★★★★★ Milne Method

★★★★★ Adams Method

These two methods are repeatedly asked in Unit-3.

2-Minute Revision

Milne

Uses 4 previous points

Predict

Correct

Improve Accuracy

Adams

Uses previous slopes

Predict

Correct

More Stable

Exam Focus

Most Expected 7-Mark Question

Explain Milne Predictor-Corrector Method.

Most Expected 14-Mark Question

Compare Milne and Adams Predictor-Corrector Methods with formulas.

One-Night Revision Shortcut

Euler → 1 slope

Modified Euler → 2 slopes

RK-4 → 4 slopes

Milne → Predict + Correct

Partial Differential Equations (PDE) – Unit 3

Mathematics-III

 RGPV Mathematics-III Unit-3 Most Important PDE Topic

 Very High Probability

Before studying **Laplace Equation, Poisson Equation, Bender-Schmidt Method, Crank-Nicolson Method, and Wave Equation**, you must first understand:

What is a Partial Differential Equation (PDE)?

What Is It?

A Partial Differential Equation (PDE) is an equation that contains **partial derivatives** of a function having **more than one independent variable**.

Simple meaning:

In Ordinary Differential Equations (ODE):

One Independent Variable

Example:

$$dy/dx=x+y$$

Here only x exists.

In PDE:

Two or More Independent Variables

Example:

$$\partial u / \partial x + \partial u / \partial y = 0$$

Here both x and y exist.

Therefore it becomes a PDE.

Why Do We Need PDE?

Many real-life engineering problems depend on:

- Position
- Time
- Temperature
- Pressure

all together.

Examples:

Heat Transfer

Temperature changes with:

Position

+

Time

Wave Motion

Wave depends on:

Distance

+

Time

Fluid Flow

Depends on:

x-direction

+

y-direction

+

Time

Therefore PDE is needed.

Real-Life Example

Suppose you put a hot iron rod in water.

Temperature changes:

- Along length

- Along width
- With time

One variable is not enough.

Hence PDE is used.

Difference Between ODE and PDE

ODE	PDE
One variable	Two or more variables
Ordinary derivative	Partial derivative
Easier	More complex
Example: dy/dx	Example: $\partial u/\partial x$
Unit-3 ODE Portion	Unit-3 PDE Portion

Basic Idea

Suppose:

Temperature

$u(x,y)$

depends on:

$x \rightarrow$ Horizontal Position

$y \rightarrow$ Vertical Position

Then:

$\partial u / \partial x$

means change in temperature along x-direction.

$\partial u / \partial y$

means change in temperature along y-direction.

Important Symbol

Ordinary Derivative

dy/dx

Partial Derivative

$\partial u / \partial x$

The symbol:

∂

(read as "partial")

is used only in PDE.

Diagram





At every point:

Temperature

Pressure

Velocity

can be different.

Types of PDE in Unit-3

1. Laplace Equation

Most Important

$$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$$

Used in:

- Heat flow
 - Electrostatics
-

2. Poisson Equation

$$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = f(x,y)$$

Used in:

- Heat sources
 - Electric fields
-

3. Heat Equation

$$\partial u / \partial t = c^2 \partial^2 u / \partial x^2$$

Used in:

- Temperature distribution

Methods:

- Bender-Schmidt
 - Crank-Nicolson
-

4. Wave Equation

$$\partial^2 u / \partial t^2 = c^2 \partial^2 u / \partial x^2$$

Used in:

- Vibrations
 - Sound waves
 - Water waves
-

Important Keywords for Exam

- Partial Differential Equation
- Partial Derivative
- Independent Variable
- Dependent Variable
- Laplace Equation
- Poisson Equation

- Heat Equation
 - Wave Equation
 - Finite Difference Method
-

Advantages

- ✓ Solves real engineering problems
 - ✓ Models heat transfer
 - ✓ Models fluid flow
 - ✓ Models wave propagation
 - ✓ Used in simulation software
-

Disadvantages

- ✗ Difficult calculations
 - ✗ Complex formulas
 - ✗ Requires numerical methods
 - ✗ Large computations
 - ✗ Computer implementation needed
-

Applications

1. Heat Transfer
2. Mechanical Engineering
3. Fluid Mechanics
4. Structural Analysis

5. Electrical Engineering
 6. Weather Prediction
 7. Aerodynamics
 8. Electromagnetic Fields
-

Frequently Asked RGPV Questions

7 Marks

1. What is PDE?
2. Difference between ODE and PDE.
3. Applications of PDE.

14 Marks

1. Explain Partial Differential Equations with examples.
 2. Explain classification and applications of PDE.
 3. Compare ODE and PDE.
-

Memory Trick

PDE = Position Dependent Equation

Remember:

ODE → One Variable

PDE → Many Variables

2-Minute Revision

1. PDE contains partial derivatives.
2. Uses symbol ∂ .
3. Depends on more than one variable.
4. Used in heat transfer.
5. Used in wave motion.
6. Used in fluid mechanics.
7. Laplace Equation is a PDE.
8. Poisson Equation is a PDE.
9. Heat Equation is a PDE.
10. Wave Equation is a PDE.
11. Most important Unit-3 topic.
12. Basis of all PDE numerical methods.

Finite Difference Solution of Two-Dimensional Laplace Equation

 **Most Important PDE Topic of Unit-3**

 Very High Probability

 RGPV Favourite Numerical Question

What Is Laplace Equation?

Laplace Equation is a Partial Differential Equation (PDE) used to find unknown values such as:

- Temperature
- Voltage
- Pressure
- Potential

at different points.

The equation is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Why Do We Need It?

Suppose we know temperature at the boundary of a metal plate.

We want to find temperature inside the plate.

Laplace Equation helps us.

Real-Life Example

Imagine a square metal plate.

Boundary temperatures:

100°C 100°C

? ? ?

? ? ?

0°C 0°C

Need:

Inside Temperatures

Laplace Equation solves this.

What is Finite Difference Method?

The differential equation is difficult to solve directly.

So we convert derivatives into algebraic equations.

This process is called:

Finite Difference Method (FDM)

Grid Representation

u1 ---- u2 ---- u3

| | |

u4 ---- u5 ---- u6

| | |

u7 ---- u8 ---- u9

Each point is called:

Mesh Point

Finite Difference Formula

For Laplace Equation:

$$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$$

becomes

$$u_{i,j} = [u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}] / 4$$

Memory Trick

Laplace Rule

Center Value

=

Average of Four Neighbours

Remember:

North

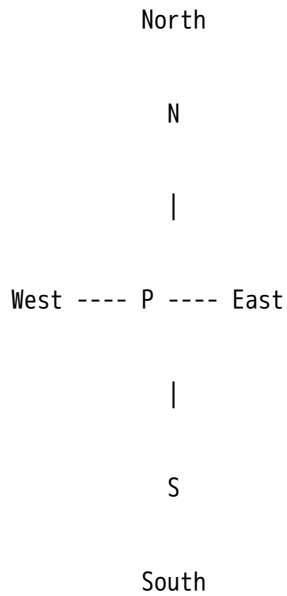
South

East

West

÷ 4

Diagram



Formula:

$$P = \frac{N + S + E + W}{4}$$

Step-by-Step Numerical

Suppose:

100	100
x	x
0	0

Let unknown values:

u_1 u_2

Using Laplace Formula:

$$u_1 = 100 + 0 + 100 + u_2/4$$

$$u_2 = 100 + 0 + u_1 + 100/4$$

Solve simultaneously.

Obtain:

$$u_1 = u_2 = 66.67$$

Advantages

- ✓ Simple method
 - ✓ Easy numerical solution
 - ✓ Computer implementation
 - ✓ High accuracy
 - ✓ Widely used
-


Applications

1. Heat transfer
2. Electrostatics
3. Fluid flow
4. Mechanical engineering
5. Structural analysis

Frequently Asked RGPV Questions

1. Solve Laplace Equation using Finite Difference Method.
 2. Derive Finite Difference Formula.
 3. Explain mesh and grid system.
 4. Applications of Laplace Equation.
-

Finite Difference Solution of Poisson Equation

 Another Favourite RGPV Topic

 Very High Probability

What Is Poisson Equation?

Poisson Equation is similar to Laplace Equation.

Difference:

Laplace:

RHS=0

Poisson:

RHS≠0

Equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

Real-Life Example

Suppose heat is being generated inside the plate.

Then:

Heat Source Exists

Laplace cannot handle this.

Poisson Equation is used.

Difference Between Laplace and Poisson

Laplace	Poisson
RHS = 0	RHS \neq 0
No source	Source present
Simpler	More complex
Heat distribution	Heat generation

Finite Difference Formula

For Poisson Equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

Finite Difference Form:

$$u_{i,j} = \frac{1}{4}(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}) - \frac{h^2}{4} f(i,j)$$

Memory Trick

Laplace vs Poisson

Laplace = No Source

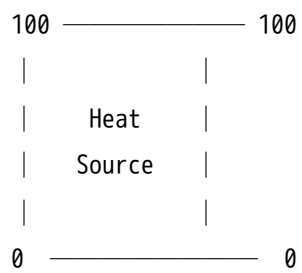
Poisson = Source Present

Remember:

Poisson = Plus Source

Diagram

Boundary Values



Poisson handles source effects.

Advantages

- ✓ Handles source terms
 - ✓ More realistic
 - ✓ Engineering applications
 - ✓ Heat generation analysis
 - ✓ Electrical field analysis
-

Applications

1. Heat generation problems
 2. Electrical engineering
 3. Fluid mechanics
 4. Electromagnetic fields
 5. Structural mechanics
-

Most Expected 7-Mark Questions

1. Explain Finite Difference Solution of Laplace Equation.
 2. Explain Poisson Equation.
 3. Difference between Laplace and Poisson Equation.
-

Most Expected 14-Mark Questions

1. Solve Two-Dimensional Laplace Equation using Finite Difference Method.
2. Solve Poisson Equation using Finite Difference Method.
3. Derive Finite Difference Formula for Laplace Equation.

2-Minute Revision

Laplace Equation

$$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$$

- ✓ No source
- ✓ Center = Average of 4 neighbours

Poisson Equation

$$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = f(x,y)$$

- ✓ Source present
- ✓ Modified finite difference formula

Exam Shortcut

Laplace → Average Formula

Poisson → Average Formula + Source Term

🔥 If only one PDE topic comes in the exam, **Laplace Equation using Finite Difference Method** is the most likely 14-mark question from Unit-3.


Implicit and explicit methods for one dimensional heat equation (BenderSchmidt and Crank-Nicholson methods),

One-Dimensional Heat Equation

Bender-Schmidt Method & Crank-Nicolson Method

 MOST IMPORTANT PDE TOPIC OF UNIT-3

 Very High Probability

 RGPV Favourite 14-Mark Question

First Understand: What is Heat Equation?

Heat Equation tells us:

How temperature changes
with
Position + Time

Suppose a metal rod is heated.

Temperature changes:

- From left to right
- With time

Heat Equation studies this change.

Mathematical Form

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

u

Temperature

x

Position

t

Time

c

Thermal Constant

Real-Life Example

Imagine:

Hot Iron Rod

Initially:

100°C ----- 100°C

After some time:


80°C ----- 80°C

After more time:

50°C ----- 50°C

Heat Equation predicts this temperature distribution.

Explicit Method (Bender-Schmidt Method)

 Most Important Theory Question



What Is Bender-Schmidt Method?

Bender-Schmidt Method is an **Explicit Finite Difference Method**.

Explicit means:

New Temperature
depends only on
Old Temperatures

No simultaneous equations required.

Basic Idea

We divide:

Rod \rightarrow Space Intervals

Time \rightarrow Time Intervals

Then create a grid.

Grid Diagram

Time \uparrow

t2 ● ● ● ●

t1 ● ● ● ●

t0 ● ● ● ●

x \rightarrow

Finite Difference Formula

For Bender-Schmidt:

When

$$\lambda = c^2 k / h^2 = 1/2$$

Formula becomes:

$$u_i^{j+1} = 1/2(u_{i-1}^j + u_{i+1}^j)$$

Memory Trick

Bender-Schmidt

Future
=
Average
of Left & Right

Remember:

$$\frac{\text{LEFT} + \text{RIGHT}}{2}$$

Working Steps

Step 1

Draw grid.

Step 2

Write boundary values.

Step 3

Apply formula.

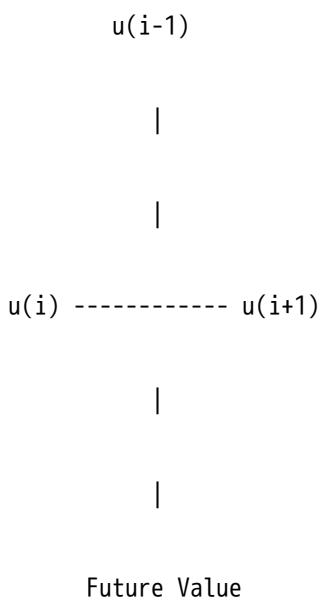
Step 4

Fill next row.

Step 5

Continue until final answer.

Diagram



Advantages

- ✓ Easy calculations
- ✓ Simple formula
- ✓ Fast method

✓ Suitable for hand calculations

✓ Frequently asked

Disadvantages

✗ Stability issue

✗ Less accurate

✗ Small time step required

✗ Error accumulation

✗ Not suitable for large intervals

Applications

1. Heat transfer
 2. Thermal analysis
 3. Mechanical engineering
 4. Metallurgy
 5. Furnace design
-

Crank-Nicolson Method

🔥 Most Important Numerical Method



What Is Crank-Nicolson Method?

Crank-Nicolson Method is an **Implicit Finite Difference Method**.

Implicit means:

Future temperatures
appear on both sides
of equation

More accurate than Bender-Schmidt.

Why Do We Need It?

Bender-Schmidt:

Simple
Less Accurate

Crank-Nicolson:

More Accurate
More Stable

Real-Life Example

Suppose weather prediction.

Bender-Schmidt:

Uses only current information.

Crank-Nicolson:

Uses:

Current Information

+

Future Trend

Hence prediction improves.

Basic Idea

Instead of using only present values,

it uses:

Present Values

+

Future Values

Therefore accuracy increases.

Crank-Nicolson Formula

$$u_i^{j+1} = u_i^j + \lambda/2 [(u_{i+1}^j - 2u_i^j + u_{i-1}^j) + (u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1})]$$

Easy Memory Trick

Crank-Nicolson

Present + Future

Remember:

Half Present

+

Half Future

Working Procedure

Step 1

Create mesh.

Step 2

Apply finite difference formula.

Step 3

Obtain simultaneous equations.

Step 4

Solve equations.

Step 5

Find temperatures.

Diagram

Present Row



Future Row



Uses both rows together.

Advantages

- ✓ Highly accurate
 - ✓ Stable
 - ✓ Less error
 - ✓ Widely used
 - ✓ Engineering favourite
-

Disadvantages

- ✗ More calculations
 - ✗ Simultaneous equations
 - ✗ Difficult manually
 - ✗ Time consuming
 - ✗ Requires matrix solution
-

Difference Between Bender-Schmidt and Crank-Nicolson

Bender-Schmidt	Crank-Nicolson
Explicit Method	Implicit Method
Simple	Complex
Less Accurate	More Accurate
Less Stable	More Stable
No simultaneous equations	Simultaneous equations required
Faster	Slower

Frequently Asked RGPV Questions

7 Marks

1. Explain Bender-Schmidt Method.
 2. Explain Crank-Nicolson Method.
 3. Difference between Explicit and Implicit Methods.
-

14 Marks

1. Solve One-Dimensional Heat Equation using Bender-Schmidt Method.
 2. Solve Heat Equation using Crank-Nicolson Method.
 3. Compare Explicit and Implicit Methods.
-

2-Minute Revision

Heat Equation

$$\partial u / \partial t = c^2 \partial^2 u / \partial x^2$$

Bender-Schmidt

Explicit Method

Uses Present Values Only

Simple

Less Accurate

Crank-Nicolson

Implicit Method

Uses Present + Future Values

More Accurate

More Stable

Exam Shortcut

Heat Equation

↓

Bender-Schmidt
(Explicit)

↓

Crank-Nicolson
(Implicit)

↓

Difference Table

RGPV 2026 Prediction:

One question from **Bender-Schmidt or Crank-Nicolson** is highly likely. Learn the formulas, difference table, and working steps—they are enough to score good marks in the PDE section.

Finite difference explicit method for wave equation.

Finite Difference Explicit Method for Wave Equation

What Is Wave Equation?

Wave Equation is a Partial Differential Equation used to study wave motion.

It tells us how a wave changes with:

- Position
- Time

Examples:

- Vibrating string
 - Sound wave
 - Water wave
 - Guitar string vibration
-

Standard Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Where:

- u = displacement of wave
 - x = position
 - t = time
 - c = wave velocity
-

Why Do We Need It?

Exact solution of wave equation can be difficult.

So we divide the string/region into small grid points and find approximate values using finite difference method.

Real-Life Example

Think of a guitar string.

When you pluck it:

- String moves up and down
- Vibration changes with time
- Shape changes at different positions

Wave equation predicts this motion.

Basic Idea

We divide:

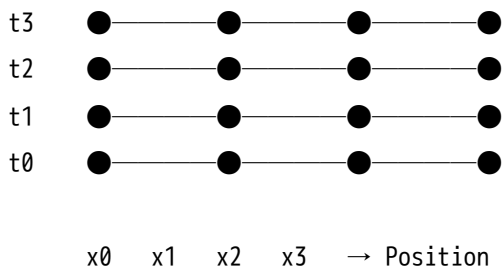
Length of string → small space intervals

Time → small time intervals

Then calculate wave displacement step-by-step.

Grid Diagram

Time ↑



Each point represents value of u .

Finite Difference Approximation

For wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Finite difference form becomes:

$$u_i^{j+1} = 2u_i^j - u_i^{j-1} + \lambda^2 [u_{i+1}^j - 2u_i^j + u_{i-1}^j]$$

Where:

$$\lambda = ck/h$$

Meaning of Symbols

- u_i^j = value at position i and time j
 - h = space interval
 - k = time interval
 - c = wave velocity
 - λ = mesh ratio
-

Special Case

If:

$$\lambda=1$$

Then formula becomes:

$$u_i(j+1)=u(i+1)^j+u(i-1)^j-u_i^{(j-1)}$$

This is very easy and commonly used in exam numericals.

Why Is It Called Explicit Method?

It is called explicit because the future value is directly calculated from known old values.

Future Value = Present Values + Previous Time Value

No simultaneous equations are required.

Step-by-Step Procedure

1. Write the wave equation.
 2. Note boundary conditions.
 3. Divide x and t into equal intervals.
 4. Find h,k,and λ .
 5. Prepare mesh/grid.
 6. Use initial conditions to fill first two rows.
 7. Apply explicit formula.
 8. Continue until required value is found.
-

Flowchart

Given Wave Equation



Create Grid



Apply Initial Conditions



Find $\lambda = ck/h$



Use Explicit Formula



Calculate Future Values

Advantages

1. Simple formula.
 2. Direct calculation.
 3. Easy to understand.
 4. No simultaneous equations.
 5. Suitable for hand calculations.
-

Disadvantages

1. Stability condition required.
 2. Approximate answer.
 3. Error may increase.
 4. More grid points mean more calculation.
 5. Not suitable for very large problems manually.
-

Applications

1. Vibrating strings
 2. Sound waves
 3. Water waves
 4. Seismic waves
 5. Mechanical vibrations
-

Difference: Heat Equation vs Wave Equation

Heat Equation	Wave Equation
First derivative in time	Second derivative in time
Temperature distribution	Wave displacement
Heat flow	Vibration/motion
Bender-Schmidt, Crank-Nicolson	Explicit finite difference
Diffusion problem	Oscillation problem

Important Keywords for Exam

- Wave Equation
 - Finite Difference Method
 - Explicit Method
 - Mesh Ratio
 - Boundary Conditions
 - Initial Conditions
 - Displacement
 - Stability
-

Frequently Asked RGPV Questions

1. Explain finite difference explicit method for wave equation.
2. Derive finite difference formula for wave equation.

3. Solve wave equation using explicit method.
 4. Write difference between heat equation and wave equation.
 5. Explain mesh/grid method for wave equation.
-

Common Mistakes Students Make

- ✗ Forgetting second time derivative.
 - ✗ Confusing heat equation formula with wave equation formula.
 - ✗ Not calculating λ .
 - ✗ Wrong use of boundary conditions.
 - ✗ Mixing h and k .
-

Memory Trick

Wave = Past + Present + Neighbours

Future value depends on:

Left neighbour

Right neighbour

Present value

Previous time value

For $\lambda=1$:

Future = Left + Right - Previous

2-Minute Revision

1. Wave equation studies vibration.
2. Standard form: $u_{tt} = c^2 u_{xx}$
3. Used for vibrating string and waves.
4. Explicit method gives future value directly.
5. Formula uses present and previous time rows.
6. h = space step.
7. k = time step.
8. $\lambda = ck/h$
9. If $\lambda = 1$, formula becomes simple.
10. No simultaneous equations required.
11. Boundary and initial conditions are important.
12. Common PDE numerical topic.

Exam Focus

★★★★ High Probability

Most Important Formula:

$$u_i^{(j+1)} = 2u_i^{(j)} - u_i^{(j-1)} + \lambda^2 (u_{i+1}^{(j)} - 2u_i^{(j)} + u_{i-1}^{(j)})$$