UNIT - 02 M3 NOTES WITH EASY EXPLAINATION

Numerical Integration

What Is It?

Numerical Integration is a method used to find the approximate value of an integral when exact integration is difficult or impossible.

Integration means finding the **area under a curve**.

In numerical integration, we calculate this area using numerical formulas instead of direct integration.

Why Do We Need It?

Many engineering problems involve functions that cannot be integrated easily.

Example:

limit from 0 to 1 ∫e^-x2dx

This integral cannot be solved by ordinary integration methods.

So we use numerical integration methods.

Real-Life Example

Suppose you want to find the area of an irregular lake.

Since the shape is not regular, exact area is difficult.

So we divide it into small sections and add the areas.

Numerical integration works in the same way.

Basic Idea

Area under a curve:

```
      Curve
       /\
      /  \
     /    \
----/------\---------
```

We divide the area into small parts.

Then add the areas of all small parts.

This gives approximate integration.

Numerical Integration Methods in RGPV

🔴 Most Important

1. Trapezoidal Rule

2. Simpson's 1/3 Rule

3. Simpson's 3/8 Rule

These three methods are repeatedly asked in RGPV exams.

Classification

```
Numerical Integration
         |
         |
    -----------------
    |       |       |
Trapezoidal  Simpson 1/3  Simpson 3/8
```

Trapezoidal Rule and Simpson's 1/3 & 3/8 Rules

RGPV Mathematics-III | One-Night Exam Notes

This is one of the **most important topics of Unit-2**.

Questions from these rules are asked almost every year in RGPV.

---

1. Trapezoidal Rule

What Is It?

Trapezoidal Rule is a numerical integration method used to find the approximate value of an integral.

It assumes the curve between two points is a straight line.

The area under the curve is divided into trapeziums.

Why Do We Need It?

Sometimes integration is difficult.

Example:

limit from 0 to 1 $\int 1/1+x^2dx$

Instead of solving exactly, we approximate the area.

Real-Life Example

Suppose a farmer wants to find the area of an irregular field.

He divides it into trapezium-shaped sections and adds all areas.

This is exactly what Trapezoidal Rule does.

Basic Idea

Replace the curve by straight lines.

Find area of each trapezium.

Add all trapezium areas.

Diagram

```
Curve

     /\
    /  \
   /    \
  /      \
 /________\
```

```
Area divided into trapeziums
```

Formula

For n equal intervals:

limit from a to b $\int y dx = h2[(y0+yn)+2(y1+y2+y3+\cdots+yn-1)]$

Where:

h = interval width

$h=b-a/n$

Step-by-Step Procedure

Step 1

Find interval width

h=b−anh=\frac{b-a}{n}h=nb−a

Step 2

Prepare x and y table.

Step 3

Substitute values into formula.

Step 4

Calculate answer.

Advantages

1. Very easy method.
2. Simple calculations.
3. Useful for tabular data.
4. Works for any interval.
5. Frequently asked in exams.

Disadvantages

1. Less accurate.
2. Assumes straight line.
3. Large error for curved functions.
4. Approximate answer.
5. Not best for high accuracy.

Memory Trick

Trapezoidal

```
First + Last once
Middle twice
```

Frequently Asked RGPV Questions

1. Derive Trapezoidal Rule.
2. Evaluate integral using Trapezoidal Rule.
3. Compare Trapezoidal and Simpson Rule.

Exam Focus

⭐⭐⭐⭐⭐ Very High Probability

---

2. Simpson's 1/3 Rule

What Is It?

Simpson's 1/3 Rule is a more accurate numerical integration method.

Instead of straight lines, it uses parabolic curves.

Why Do We Need It?

Trapezoidal Rule gives rough approximation.

Simpson's Rule fits the curve more accurately.

Hence error decreases.

Real-Life Example

Suppose you want to draw a road.

Using straight lines gives poor shape.

Using a smooth curve gives better shape.

That is Simpson's idea.

Basic Idea

Replace curve with parabola.

Calculate area under parabola.

Diagram

```
Parabolic Curve

        )
      )
    )
----------------
```

Condition

Number of intervals must be EVEN.

Example:

```
n = 2 ✓

n = 4 ✓

n = 6 ✓

n = 5 ✗
```

Formula

limit from a to b $\int y dx = \frac{h}{3}[(y_0+y_n)+4(y_1+y_3+y_5+\cdots)+2(y_2+y_4+y_6+\cdots)]$

Pattern

Remember:

```
1 4 2 4 2 4 1
```

First and last → 1

Odd terms → 4

Even terms → 2

Step-by-Step Procedure

Step 1

Find h.

Step 2

Calculate y values.

Step 3

Apply pattern:

```
1 4 2 4 2 4 1
```

Step 4

Multiply by

h/3

Advantages

1. More accurate.
2. Less error.
3. Suitable for curved graphs.
4. Most important RGPV topic.
5. Easy pattern.

Disadvantages

1. Intervals must be even.
2. Longer formula.
3. More calculations.
4. Approximate answer.
5. Mistakes possible.

Memory Trick

Simpson 1/3

```
1 4 2 4 2 4 1
```

Think:

```
Odd = 4
```

```
Even = 2
```

Frequently Asked RGPV Questions

1. Derive Simpson's 1/3 Rule.
2. Evaluate integral using Simpson's 1/3 Rule.
3. Difference between Trapezoidal and Simpson Rule.

Exam Focus

⭐⭐⭐⭐⭐ Very High Probability

---

3. Simpson's 3/8 Rule

What Is It?

Simpson's 3/8 Rule is another numerical integration method.

It is used when intervals are multiples of 3.

Why Do We Need It?

Sometimes interval conditions do not suit Simpson's 1/3 Rule.

Then Simpson's 3/8 Rule is used.

Basic Idea

Uses cubic approximation.

Gives good accuracy.

Diagram

```
Smooth Cubic Curve

      )
    )
  )
)
-----------------
```

Condition

Number of intervals must be multiple of 3.

Examples:

```
n=3 ✓

n=6 ✓

n=9 ✓

n=4 ✗
```

Formula

limit from a to b $\int y dx = 3h/8[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots)+2(y_3+y_6+y_9+\cdots)]$

Pattern

Remember:

1 3 3 2 3 3 2 3 3 1

Step-by-Step Procedure

Step 1

Find h.

Step 2

Check intervals are multiple of 3.

Step 3

Apply pattern.

Step 4

Multiply by

3h8\frac{3h}{8}83h

Advantages

1. More accurate.
2. Suitable for cubic curves.
3. Good approximation.
4. Useful in engineering.
5. Important numerical method.

Disadvantages

1. Intervals must be multiple of 3.
2. Long calculations.
3. Easy to make mistakes.
4. Approximate answer.
5. Not as commonly used as 1/3 Rule.

Memory Trick

Simpson 3/8

```
1 3 3 2 3 3 2 3 3 1
```

Repeat:

```
3 3 2
3 3 2
3 3 2
```

Frequently Asked RGPV Questions

1. Derive Simpson's 3/8 Rule.
2. Evaluate integral using Simpson's 3/8 Rule.
3. Compare Simpson's 1/3 and 3/8 Rules.

Exam Focus

⭐⭐⭐⭐⭐ Very High Probability

Difference Between All Three Rules

Feature	Trapezoidal	Simpson 1/3	Simpson 3/8
Curve Used	Straight Line	Parabola	Cubic Curve
Accuracy	Low	High	Very High
Condition	Any n	Even n	Multiple of 3
Formula Factor	h/2	h/3	3h/8
Exam Importance	⭐⭐⭐⭐⭐	⭐⭐⭐⭐⭐	⭐⭐⭐⭐

Last 30-Second Revision

```
Trapezoidal:
Factor = h/2
Pattern = 1 2 2 2 1

Simpson 1/3:
Factor = h/3
Pattern = 1 4 2 4 2 4 1
Even intervals compulsory

Simpson 3/8:
Factor = 3h/8
Pattern = 1 3 3 2 3 3 2 3 3 1
Intervals multiple of 3 compulsory
```

Gauss Elimination Method

(Solution of Simultaneous Linear Algebraic Equations)

🔥 **RGPV Exam Point of View: VERY IMPORTANT TOPIC**

⭐⭐⭐⭐⭐ Frequently Asked in 7 Marks and 14 Marks

What Is It?

Gauss Elimination Method is a numerical method used to solve simultaneous linear equations.

Simultaneous equations means many equations having many unknown variables.

Example:

```
2x + y + z = 8

3x + 2y + 3z = 15

x + y + 2z = 8
```

Gauss Elimination converts these equations into a simpler form and then finds x, y, z.

Why Do We Need It?

When there are many equations, solving manually becomes difficult.

Gauss Elimination provides a systematic procedure.

Used extensively in:

- Engineering
- Computer Science
- Machine Learning
- Circuit Analysis
- Structural Design

Real-Life Example

Imagine three friends bought pens, notebooks and pencils.

You know total prices but not individual prices.

You create equations.

Gauss Elimination helps find individual prices.

Basic Idea

The method has two steps:

Step 1

Convert equations into:

```
Upper Triangular Form
```

Example:

```
a11 a12 a13

0   a22 a23

0   0   a33
```

Step 2

Use Back Substitution.

Find:

```
z first

then y

then x
```

Flowchart

```
Given Equations
       ↓
Matrix Form
       ↓
Eliminate Lower Terms
       ↓
Upper Triangular Matrix
       ↓
Back Substitution
       ↓
Solution
```

Step-by-Step Procedure

Suppose:

```
2x + y + z = 8

3x + 2y + 3z = 15

x + y + 2z = 8
```

Step 1

Write Augmented Matrix

```
2   1   1 | 8

3   2   3 |15

1   1   2 | 8
```

Step 2

Make first column below pivot zero.

Pivot:

```
2
```

Eliminate:

```
R2 = R2 - (3/2)R1
```

```
R3 = R3 - (1/2)R1
```

New Matrix

```
2   1   1 | 8

0  0.5 1.5 | 3

0  0.5 1.5 | 4
```

Step 3

Eliminate second column below pivot.

Pivot:

```
0.5
```

Operation:

```
R3 = R3 - R2
```

Result:

```
2   1   1 | 8

0 0.5 1.5 | 3

0 0   0 | 1
```

This tells system is inconsistent.

No solution exists.

Numerical Example (Important)

Solve:

```
x + y + z = 6

2x + 3y + z = 10

x + 2y + 3z = 14
```

Step 1

Matrix:

```
1 1 1 | 6

2 3 1 |10

1 2 3 |14
```

Step 2

Make first column zero.

```
R2 = R2 - 2R1

R3 = R3 - R1
```

Result:

```
1 1 1 | 6

0 1 -1 | -2

0 1 2 | 8
```

Step 3

Make second column zero.

```
R3 = R3 - R2
```

Result:

```
1 1 1 | 6

0 1 -1 | -2
```

```
0 0 3 |10
```

Back Substitution

Third equation:

```
3z = 10
```

```
z=10/3
```

Second equation:

```
y - z = -2
```

```
y=−2+10/3
y= 4/3
```

First equation:

```
x+y+z=6
```

```
x+4/3+10/3=6
x=4/3
```

Final Answer

```
x=4/3, y=4/3, z=10/3
```

Important Keywords for Exam

- Simultaneous Equations
- Matrix
- Augmented Matrix
- Pivot Element
- Row Operations
- Upper Triangular Matrix
- Back Substitution

Advantages

1. Systematic method.
2. Easy to program.
3. Suitable for large equations.
4. Widely used in engineering.
5. Accurate results.

Disadvantages

1. Long calculations.
2. Round-off errors possible.
3. Pivot may become zero.
4. Time consuming manually.
5. Not ideal for huge systems by hand.

Difference: Gauss Elimination vs Gauss Jordan

Gauss Elimination	Gauss Jordan
Creates upper triangular matrix	Creates identity matrix
Uses back substitution	No back substitution
Less calculations	More calculations
Faster	Slower
Most used	Used for inverse matrix

Frequently Asked RGPV Questions

7 Marks

1. Explain Gauss Elimination Method.
2. Solve equations using Gauss Elimination.
3. What is back substitution?

---

14 Marks

1. Explain Gauss Elimination Method with algorithm.
2. Solve a system of three equations using Gauss Elimination.
3. Compare Gauss Elimination and Gauss Jordan.

Common Mistakes Students Make

❌ Forgetting row operations

❌ Arithmetic mistakes

❌ Wrong elimination factor

❌ Back substitution errors

❌ Wrong sign while subtracting rows

Memory Trick

G E B

```
G → Given Matrix

E → Eliminate Lower Terms

B → Back Substitute
```

Remember:

Gauss = Eliminate → Triangular → Back Substitute

2-Minute Revision

1. Used to solve simultaneous equations.
2. Convert equations into matrix form.
3. Make lower elements zero.
4. Obtain upper triangular matrix.
5. Use row operations.
6. Apply back substitution.
7. Find z first.
8. Then y.
9. Then x.

10. Very important RGPV topic.

Exam Focus

⭐⭐⭐⭐⭐ Very High Probability

Most Important Diagram

```
a11  a12  a13

0    a22  a23

0     0   a33
```

Solution of Simultaneous Linear Algebraic Equations by Gauss Elimination Method

(RGPV One-Night Preparation Notes)

🔥 **Most Important Topic of Module-2**

⭐⭐⭐⭐⭐ Very High Probability

What Are Simultaneous Linear Algebraic Equations?

When two or more linear equations contain the same unknown variables and must be solved together, they are called **simultaneous linear algebraic equations**.

Example

```
x+y+z=6
2x+3y+z=10
x+2y+3z=14

Here:
```

- x, y, z = unknown variables
- Three equations must be solved together.

What Is Gauss Elimination Method?

Gauss Elimination Method is a numerical method used to solve simultaneous linear equations.

The main idea is:

Step 1

Convert equations into **Upper Triangular Form**

```
[a11 a12 a13
  0  a22 a23
  0   0  a33]
```

Step 2

Use **Back Substitution**

Find:

```
z first
y second
x last
```

Why Do We Need It?

Suppose there are:

- 10 equations
- 10 unknowns

Normal algebra becomes difficult.

Gauss Elimination gives a systematic procedure.

Used in:

- Circuit Analysis
- Machine Learning
- Structural Engineering
- Computer Graphics
- Numerical Computation

Real-Life Example

Suppose:

- Pen price = x
- Notebook price = y
- Pencil price = z

You know total bills from three shops.

You form three equations.

Gauss Elimination helps find x, y and z.

Algorithm of Gauss Elimination

```
Given Equations
        ↓
Convert to Matrix
        ↓
Make Lower Terms Zero
        ↓
Upper Triangular Matrix
        ↓
Back Substitution
        ↓
Find Unknown Values
```

Numerical Example

Solve:

```
x+y+z=6
```

```
2x+3y+z=10
```

```
x+2y+3z=14
```

Step 1: Write Augmented Matrix

```
[111 |6
 231 |10
 123 |14]
```

Step 2: Eliminate First Column

Perform:

```
R2=R2−2R1
R3=R3−R1
```

New matrix:

```
[111 |6
 01−1 |−2
 012  |8]
```

Step 3: Eliminate Second Column

Perform:

```
R3=R3−R2
```

New matrix:

```
[111  |6
 01−1 |−2
 003  |10]
```

Now Upper Triangular Form is obtained.

Back Substitution

From Third Equation

```
3z=10 z=10/3
```

From Second Equation

```
y−z=−2
```

Substitute z:

```
y−10/3=−2
```

```
y=3/4
```

From First Equation

```
x+y+z=6
```

Substitute:

```
x+4/3+10/3=6
 x=4/3
```

Final Answer

```
x=4/3 , y=4/3, z=10/3
```

Important Terms

Augmented Matrix

Matrix containing coefficients and constants.

Pivot Element

Element used to eliminate lower elements.

Row Operation

Operation performed on rows.

Upper Triangular Matrix

Matrix where all elements below diagonal are zero.

Back Substitution

Finding variables starting from last equation.

Advantages

1. Systematic procedure.
2. Suitable for large systems.
3. Accurate method.
4. Easy for computer implementation.
5. Frequently used in engineering.

Disadvantages

1. Long calculations.
2. Arithmetic mistakes possible.
3. Pivot may become zero.
4. Round-off errors.
5. Time consuming manually.

Memory Trick

G-E-B Rule

```
G = Given Matrix

E = Eliminate Lower Elements

B = Back Substitute
```

Remember:

Gauss = Eliminate → Triangular → Back Substitute

Gauss-Jordan Method

(Solution of Simultaneous Linear Algebraic Equations)

🔥 **RGPV Mathematics-III Module-2 Important Topic**

⭐⭐⭐⭐ High Probability

What Is Gauss-Jordan Method?

Gauss-Jordan Method is a numerical method used to solve simultaneous linear equations.

It is an extension of Gauss Elimination Method.

In Gauss Elimination:

```
Upper Triangular Matrix
```

is obtained.

In Gauss-Jordan:

```
Identity Matrix
```

is obtained.

Why Do We Need It?

Instead of using back substitution, Gauss-Jordan directly gives the solution.

Used in:

- Numerical Analysis
- Matrix Inversion
- Computer Programming
- Engineering Calculations

Real-Life Example

Suppose:

You have three unknown prices:

```
Pen = x
Notebook = y
Pencil = z
```

Three bills give three equations.

Gauss-Jordan directly finds all prices.

Basic Idea

Convert matrix into:

[1 0 0

0 1 0

0 0 1]

called **Identity Matrix**.

Then solution is obtained directly.

Difference Between Gauss Elimination and Gauss Jordan

Gauss Elimination	Gauss Jordan
Upper triangular matrix obtained	Identity matrix obtained
Back substitution needed	No back substitution
Less calculations	More calculations
Faster	Slower
More common	Used for inverse matrix also

Algorithm

```
Given Equations
       ↓
Convert into Matrix
       ↓
Make Diagonal Elements = 1
       ↓
Make All Other Elements = 0
       ↓
Identity Matrix
       ↓
Direct Solution
```

Numerical Example

Solve:

x+y+z=6

2x+3y+z=10

x+2y+3z=14

Step 1: Write Augmented Matrix

[1 1 1 |6

2 3 1 |10

1 2 3 |14]

Step 2: Make First Column Zero

Perform:

R2=R2−2R1

R3=R3−R1

Result:

[1 1 1 |6

0 1 −1 |−2

0 1 2 |8]

Step 3: Make Second Column Zero

Perform:

R3=R3−R2

Result:

[1 1 1 |6

0 1−1 |−2

0 0 3 |10]

Step 4: Make Pivot 1

Divide Row 3 by 3:

R3=R3/3

Result:

[111 |6

01−1 |−2

001 |10/3]

Step 5: Make Above Elements Zero

For Row 2:

R2=R2+R3

For Row 1:

R1=R1−R3

Now eliminate second column from Row 1:

R1=R1−R2

Result:

[100 4/3

010 4/3

001 10/3]

Final Solution

x=4/3

y=4/3y

z=10/3

Diagram

```
Original Matrix
       ↓
Row Operations
       ↓
Identity Matrix
       ↓
Direct Solution
```

Important Keywords for Exam

- Identity Matrix
- Pivot Element
- Row Operations
- Augmented Matrix
- Simultaneous Equations
- Direct Method

Advantages

1. Direct solution.
2. No back substitution.
3. Easy to understand.
4. Useful for matrix inversion.
5. Suitable for computer implementation.

Disadvantages

1. More calculations.
2. Time consuming.
3. Arithmetic mistakes possible.
4. Not efficient for large systems manually.
5. More row operations than Gauss Elimination.

Frequently Asked RGPV Questions

7 Marks

1. Explain Gauss-Jordan Method.
2. Differentiate Gauss Elimination and Gauss-Jordan.
3. What is Identity Matrix?

14 Marks

1. Solve simultaneous equations using Gauss-Jordan Method.
2. Explain Gauss-Jordan algorithm with example.
3. Compare Gauss Elimination and Gauss-Jordan Methods.

Memory Trick

GJ = Go to Identity

```
Gauss Elimination:
Triangular Matrix

Gauss Jordan:
Identity Matrix
```

Remember:

```
Jordan = Zero Everywhere Except Diagonal
```

2-Minute Revision

1. Gauss-Jordan solves simultaneous equations.
2. It converts matrix into identity matrix.
3. No back substitution required.
4. Uses row operations.
5. Pivot elements are converted to 1.
6. Other elements are converted to 0.
7. Solution obtained directly.
8. More calculations than Gauss Elimination.
9. Important for inverse matrix.
10. Frequently asked in RGPV exams.

Crout's Method (LU Decomposition Method)

Solution of Simultaneous Linear Algebraic Equations

🔥 **RGPV Mathematics-III Module-2 Important Topic**

⭐⭐⭐⭐ Frequently Asked in 7 Marks & 14 Marks

What Is Crout's Method?

Crout's Method is a numerical method used to solve simultaneous linear equations.

Instead of solving directly, the coefficient matrix is divided into two matrices:

A=LU

Where:

L = Lower Triangular Matrix

[L11 0 0

L21 L22 0

L31   L32 L33]

U = Upper Triangular Matrix

[1 u12 u13

0   1   u23

0   0   1]

Why Do We Need It?

Suppose we have:

```
10 equations
10 unknowns
```

Direct solution becomes lengthy.

Crout's Method simplifies calculations by factorizing the matrix.

Used in:

- Engineering Analysis
- Computer Programming
- Machine Learning
- Structural Engineering
- Circuit Analysis

Real-Life Example

Suppose a big task is difficult.

You divide it into two smaller tasks.

Then solve each task separately.

Crout's Method works exactly like this.

Instead of solving A directly:

```
A = LU
```

Then solve smaller systems.

Basic Idea

Given:

AX=B

First write:

A=LU

Then:

LUX=B

Let:

UX=Y

Then:

LY=B

Now solve:

Step 1

LY=B

Find Y

Step 2

UX=Y

Find X

Flowchart

```
AX = B
   ↓
A = LU
   ↓
LY = B
   ↓
Find Y
   ↓
UX = Y
   ↓
Find X
```

Crout's Method Formula

Given:

A=[a11 a12 a13

a21 a22 a23

a31 a32 a33]

Assume:

A=LU

where

L=[L11 0 0

L21 L22 0

L31 L32 L33]

and

U=[1 u12 u13

0 1 u23

0 0 1]

Numerical Example

Solve:

2x+y+z=5

4x−6y=−2

−2x+7y+2z=9

Step 1: Matrix Form

A=[2 1 1

4−60

−272]

B=5

-2

9

Step 2: Assume

A=LU

Compare coefficients.

First Column

$L_{11}=2$

L_{21}

$L_{31}=-2$

First Row of U

$u_{12}=1/2$

$u_{13}=1/2$

Second Diagonal

$L_{22}=-6-(4)(0.5)$

$L_{22}=-8$

Second Row

$u_{23}=0-(4)(0.5)/-8$

$u_{23}=0.25$

Third Row

$L_{32}=7-(-2)(0.5)$

$L_{32}=8$

$L_{33}=2-(-2)(0.5)-8(0.25)$

$L_{33}=1$

Final L Matrix

L=[2 0 0

4−8 0

−2 8 1]

Final U Matrix

U=[1 0.5 0.5

0 1 0.25

0 0 1]

Step 3: Solve LY = B

LY=B

[2 0 0

4−8 0

−2 8 1][y1 y2 y3]=[5 −2 9]

Result:

y1=2.5 y2=1.5 y3=2

Step 4: Solve UX = Y

UX=Y

[1 0.5 0.5

 0   1   0.25

 0   0   1][xyz]=[2.5 1.5 2]

From last equation:

z=2

Second equation:

y+0.25(2)=1.5

y=1

First equation:

x+0.5(1)+0.5(2)=2.5

x=1

Final Solution

x=1

y=1

z=2

Important Keywords for Exam

- LU Decomposition
- Lower Triangular Matrix
- Upper Triangular Matrix
- Matrix Factorization

- Forward Substitution
- Backward Substitution

Advantages

1. Reduces lengthy calculations.
2. Useful for repeated solutions.
3. Efficient for computer programming.
4. Suitable for large systems.
5. Accurate method.

Disadvantages

1. Difficult for beginners.
2. Long matrix calculations.
3. Arithmetic errors possible.
4. Requires matrix knowledge.
5. More complex than Gauss Elimination.

Difference: Gauss Elimination vs Crout Method

Gauss Elimination	Crout Method
Direct elimination	LU factorization
Upper triangular matrix	L and U matrices
Back substitution	Forward + Back substitution
Easier	More systematic
Manual solution common	Computer solution common

Frequently Asked RGPV Questions

7 Marks

1. Explain Crout's Method.
2. What is LU decomposition?
3. Define Lower and Upper Triangular Matrix.

14 Marks

1. Solve simultaneous equations using Crout's Method.
2. Explain LU factorization with example.
3. Compare Crout and Gauss Elimination Methods.

Memory Trick

C-L-U Rule

```
Crout
   ↓
L Matrix
   ↓
U Matrix
   ↓
Solve
```

Remember:

```
Crout = Cut Matrix into L and U
```

2-Minute Revision

1. Crout's Method solves simultaneous equations.
2. It uses LU decomposition.
3. Matrix A is divided into L and U.
4. First solve LY = B.
5. Then solve UX = Y.
6. L = Lower Triangular Matrix.
7. U = Upper Triangular Matrix.
8. Used in numerical analysis.
9. Important RGPV topic.
10. Frequently asked in long questions.

Jacobi Method and Gauss-Seidel Method

Iterative Methods for Solving Simultaneous Linear Equations

🔥 **RGPV Mathematics-III Module-2 Most Important Topic**

⭐⭐⭐⭐⭐ Very High Probability

These methods are frequently asked in:

- 7 Marks Theory
- 14 Marks Numerical
- Viva Questions

Why Do We Need Iterative Methods?

Suppose there are:

```
100 Equations
100 Unknowns
```

Gauss Elimination becomes lengthy.

So we use iterative methods.

Iterative Method

Means:

```
Guess
↓
Improve Guess
↓
Improve Again
↓
Continue Until Correct Answer
```

Difference Between Direct and Iterative Methods

Direct Method	Iterative Method
Gives answer directly	Gives answer gradually
Example: Gauss Elimination	Example: Jacobi
More calculations	Less memory
Fast for small systems	Better for large systems

Jacobi Method

What Is It?

Jacobi Method is an iterative numerical method used to solve simultaneous linear equations.

Each new value is calculated using the OLD values of other variables.

Real-Life Example

Suppose three students estimate marks.

After seeing friends' estimates, they update their own estimate.

This process repeats until everyone gets close to the actual marks.

This is Jacobi Method.

Basic Idea

Given:

```
10x+y+z=12
x+10y+z=12
x+y+10z=12
```

Rewrite:

```
x=12−y−z/10
y=12−x−z/10
z=12−x−y/10
```

Algorithm

Step 1

Assume initial values

```
x=0, y=0, z=0
```

Step 2

Find new values.

```
x1=12−0−0/10
x1=1.2
```

```
y1=12−0−0/10
y1=1.2
```

```
z1=12−0−0/10
z1=1.2
```

Iteration Table

Iteration	x	y	z
0	0	0	0
1	1.2	1.2	1.2
2	0.96	0.96	0.96
3	1.008	1.008	1.008

Continue until values stop changing.

Jacobi Flowchart

```
Guess Values
      ↓
```

```
Calculate New Values
       ↓
Use OLD Values
       ↓
Check Error
       ↓
Repeat
```

Advantages of Jacobi

1. Simple method.
2. Easy to program.
3. Suitable for large systems.
4. Less storage needed.
5. Good for computers.

Disadvantages of Jacobi

1. Slow convergence.
2. More iterations required.
3. Not suitable for all matrices.
4. Accuracy depends on initial guess.
5. Slower than Gauss-Seidel.

Memory Trick

Jacobi

```
J = Just Old Values
```

Remember:

Jacobi always uses OLD values.

Gauss-Seidel Method

What Is It?

Gauss-Seidel Method is an improved version of Jacobi Method.

It uses newly calculated values immediately.

Therefore it converges faster.

Real-Life Example

Suppose three friends are solving a puzzle.

As soon as one friend finds a better answer, the others use it immediately.

This saves time.

That is Gauss-Seidel Method.

Basic Idea

Same equations:

```
10x+y+z=12
x+10y+z=12
x+y+10z=12
```

Rewrite:

```
x=12−y−z/10
y=12−x−z/10
z=12−x−y/10
```

Step-by-Step

Assume:

```
x=0, y=0, z=0
```

First Iteration

Calculate x

```
x1=12−0−0/10
x1=1.2
```

Calculate y

Now use NEW x

```
y1=12−1.2−0 / 10
y1= 1.08
```

Calculate z

Use NEW x and NEW y

```
z1=12−1.2−1.08/10
z1=0.972
```

Observe:

Gauss-Seidel immediately uses new values.

This makes it faster.

Flowchart

```
Guess Values
      ↓
Find x
      ↓
Use New x
      ↓
Find y
      ↓
Use New x & y
      ↓
Find z
      ↓
Repeat
```

Advantages of Gauss-Seidel

1. Faster convergence.
2. Less iterations.
3. More efficient.
4. Better than Jacobi.
5. Widely used in engineering.

Disadvantages

1. Slightly complex.
2. Not suitable for all matrices.
3. Divergence possible.
4. Initial guess required.
5. Manual calculations can be lengthy.

Difference Between Jacobi and Gauss-Seidel

Jacobi	Gauss-Seidel
Uses old values	Uses new values immediately
Slower	Faster
More iterations	Less iterations
Easier	Slightly difficult
Less efficient	More efficient

Important Keywords for Exam

Jacobi

- Iterative Method
- Old Values
- Approximation
- Convergence
- Successive Iterations

Gauss-Seidel

- Iterative Method
- New Values
- Fast Convergence
- Numerical Solution
- Approximation

Frequently Asked RGPV Questions

7 Marks

1. Explain Jacobi Method.
2. Explain Gauss-Seidel Method.
3. Difference between Jacobi and Gauss-Seidel.

---

14 Marks

1. Solve simultaneous equations using Jacobi Method.
2. Solve simultaneous equations using Gauss-Seidel Method.
3. Compare Jacobi and Gauss-Seidel Methods.

PYQ Analysis

🔴 MUST STUDY

⭐⭐⭐⭐⭐ Jacobi Method

⭐⭐⭐⭐⭐ Gauss-Seidel Method

Frequently Asked

- Algorithm of Jacobi
- Algorithm of Gauss-Seidel
- Numerical Problems
- Comparison Table

Relaxation Method (Successive Over-Relaxation - SOR Method)

Numerical Solution of Simultaneous Linear Equations

🔥 RGPV Mathematics-III Module-2 Important Topic

⭐⭐⭐⭐ High Probability

Usually asked as:

- Explain Relaxation Method.
- Difference between Gauss-Seidel and Relaxation Method.
- Advantages of Relaxation Method.
- Numerical using Relaxation Method.

What Is Relaxation Method?

Relaxation Method is an improved version of the **Gauss-Seidel Method**.

It speeds up convergence (finding the answer faster).

Instead of accepting the new value directly, we adjust it using a **Relaxation Factor (ω)**.

Why Do We Need It?

Suppose Gauss-Seidel takes:

```
20 Iterations
```

to reach the answer.

Relaxation Method may require:

```
8-10 Iterations
```

Therefore it saves time and computations.

Real-Life Example

Imagine you are travelling from Gwalior to Bhopal.

Gauss-Seidel

Travels at normal speed.

Relaxation Method

Travels slightly faster and reaches destination earlier.

That extra speed is called:

```
Relaxation Factor (ω)
```

Basic Idea

Gauss-Seidel gives:

Xnew

Relaxation Method modifies it as:

$X^{(k+1)}=X^{(k)}+\omega(X_{GS}-X^{(k)})$

where:

ω (Omega)

Relaxation Factor

XGS

Gauss-Seidel Value

X(k)

Old Value

Important Formula

$X_{new}=X_{old}+\omega(X_{GS}-X_{old})$

Meaning of ω

Case 1

$\omega=1$

Normal Gauss-Seidel Method

Case 2

$\omega>1$

Over Relaxation

Fast convergence

Case 3

$\omega<1$

Under Relaxation

Slow convergence

Diagram

```
Initial Guess
      ↓
Gauss-Seidel Value
      ↓
Apply ω
      ↓
Improved Value
      ↓
Repeat
```

Step-by-Step Procedure

Suppose equation:

$10x+y=11$

$x+10y=11$

Initial Guess

$x=0, y=0$

Gauss-Seidel Value

$x_{GS}=11-y/10$

$x_{GS}=1.1$

Suppose:

$\omega=1.2$

Relaxed Value

$x_{new}=0+1.2(1.1-0)$

$x_{new}=1.32x$

This value moves faster toward the actual solution.

Types of Relaxation

1. Under Relaxation

$0<\omega<1$

Example:

$\omega=0.8$

Used when solution oscillates.

---

2. Over Relaxation

$1<\omega<2$

Example:

$\omega=1.3$

Most commonly used.

Provides faster convergence.

Flowchart

```
Guess Values
        ↓
Apply Gauss-Seidel
        ↓
Apply Relaxation Factor
        ↓
New Approximation
        ↓
Check Error
        ↓
Repeat
```

Advantages

1. Faster than Gauss-Seidel.
2. Less iterations required.
3. Better convergence.
4. Saves computation time.

5. Useful for large systems.

Disadvantages

1. Choosing ω is difficult.
2. Wrong ω may diverge.
3. More calculations.
4. Not suitable for every problem.
5. Requires experience.

Difference Between Gauss-Seidel and Relaxation

Gauss-Seidel	Relaxation
Uses new values	Uses new values + ω
Normal speed	Faster speed
ω = 1	ω > 1
More iterations	Fewer iterations
Simpler	Slightly complex

Important Keywords for Exam

- Relaxation Factor
- Omega (ω)
- Over Relaxation
- Under Relaxation
- Convergence
- Iterative Method
- SOR Method

Frequently Asked RGPV Questions

7 Marks

1. Explain Relaxation Method.
2. What is Relaxation Factor?
3. Difference between Gauss-Seidel and Relaxation Method.

---

14 Marks

1. Explain Successive Over Relaxation Method.
2. Derive Relaxation Formula.
3. Compare Jacobi, Gauss-Seidel and Relaxation Methods.

Common Mistakes Students Make

❌ Forgetting value of ω

❌ Using ω less than 0

❌ Confusing with Gauss-Seidel

❌ Wrong substitution

❌ Not understanding convergence

Memory Trick

OMEGA Rule

```
ω = Speed Booster
```

Remember:

```
Gauss-Seidel + ω
=
Relaxation Method
```

PYQ Analysis

⭐⭐⭐⭐ High Probability

Frequently Asked Topics:

- Definition of Relaxation Method
- Relaxation Factor
- Over Relaxation
- Under Relaxation
- Comparison with Gauss-Seidel

2-Minute Revision

1. Relaxation Method is an improved Gauss-Seidel Method.
2. Uses Relaxation Factor (ω).
3. Formula:

$X_{new} = X_{old} + \omega(X_{GS} - X_{old})$

4. ω = 1 → Gauss-Seidel.
5. ω > 1 → Over Relaxation.

6. $\omega < 1$ → Under Relaxation.
7. Faster convergence.
8. Fewer iterations.
9. Useful for large equations.
10. Important RGPV theory question.

Exam Focus

⭐⭐⭐⭐ High Probability

Most Important Definition

Relaxation Method is an iterative numerical technique used to solve simultaneous linear equations by accelerating the convergence of the Gauss-Seidel Method using a relaxation factor (ω).

Most Important Formula

$X_{new} = X_{old} + \omega(X_{GS} - X_{old})$

Most Expected 7-Mark Question

Explain Relaxation Method and Relaxation Factor.