UNIT -01 DETAILED AND EASY NOTES

BISECTION METHOD (Solution of Polynomial and Transcendental Equations)

⭐⭐⭐⭐⭐ **Most Important Topic of Module-1**

(RGPV Favourite Numerical + Theory Question)

What Is Bisection Method?

Bisection Method is a numerical method used to find the root of an equation.

A **root** means the value of x for which:

f(x)=0

The method repeatedly divides the interval into two equal parts until the root is found.

Why Do We Need It?

Many equations cannot be solved easily by algebra.

Example:

x^3-x-1=0

Finding exact root is difficult.

So numerical methods like Bisection Method are used.

Real-Life Example

Suppose you lost a key in a 100-meter road.

Instead of searching the whole road, you check the middle first.

Then again check the middle of the remaining part.

This process continues.

This is exactly how Bisection Method works.

Basic Idea

Find two values:

a and b

such that

f(a) and f(b) have opposite signs.

Meaning:

```
f(a) > 0

f(b) < 0
```

or

```
f(a) < 0

f(b) > 0
```

Then a root definitely exists between them.

Now repeatedly divide interval into two equal halves.

Important Condition

Before applying Bisection Method:

```
f(a) × f(b) < 0
```

Must be satisfied.

This proves root lies between a and b.

Step-by-Step Algorithm

Step 1

Given equation:

```
f(x)=0
```

Step 2

Choose two values:

a and b

such that

```
f(a) × f(b) < 0
```

Step 3

Calculate midpoint

c=a+b/2

Step 4

Find value of

f(c)

Step 5

Check sign

If

```
f(a) × f(c) < 0
```

then root lies between

a and c

Otherwise root lies between

c and b

Step 6

Repeat process until required accuracy is obtained.

Flowchart

```
Start
   ↓
Choose a,b
   ↓
f(a)f(b)<0 ?
   ↓
Calculate c=(a+b)/2
   ↓
Find f(c)
   ↓
Select new interval
   ↓
Repeat
   ↓
Root Found
```

Numerical Example

Find root of

x^3-x-2=0

Step 1

Let

x=1

```
f(1)=1-1-2=-2
```

---

x=2

```
f(2)=8-2-2=4
```

Since

```
(-2)(4)<0
```

Root lies between 1 and 2.

Step 2

Find midpoint

c=1+2/2=1.5

Step 3

Calculate

```
f(1.5)

=3.375−1.5−2

=-0.125
```

Negative

Root lies between

1.5 and 2

Step 4

New midpoint

c=1.5+2/2=1.75

Continue until desired accuracy.

Diagram

```
1            2
|------------|

      1.5
       |

Root lies here
```

Important Formula

Midpoint Formula

c=a+b/2

Advantages

✅ Very simple method

✅ Easy to understand

✅ Guaranteed convergence

✅ No derivative required

✅ Suitable for beginners

✅ Highly accurate

Disadvantages

❌ Slow process

❌ Many iterations needed

❌ Works only if root is bracketed

❌ Not efficient for complex equations

❌ Requires sign change

Applications

✅ Engineering calculations

✅ Numerical Analysis

✅ Root finding

✅ Scientific Computing

✅ Computer Algorithms

✅ Mathematical Modeling

Difference Between Bisection and Newton-Raphson

Bisection	Newton-Raphson
Slow	Fast
No derivative	Derivative required
Always converges	May diverge
Easy	Slightly difficult
Beginner friendly	Advanced

Important Keywords for Exam

- Root of Equation
- Numerical Method
- Interval Halving
- Convergence
- Approximation
- Polynomial Equation
- Transcendental Equation
- Midpoint

Frequently Asked RGPV Questions

7 Marks

1. Explain Bisection Method.
2. Write algorithm of Bisection Method.
3. State advantages and disadvantages.

14 Marks

1. Explain Bisection Method with numerical example.
2. Derive Bisection Method algorithm.
3. Compare Bisection and Newton-Raphson Methods.

Common Mistakes Students Make

❌ Forgetting sign condition

```
f(a)f(b)<0
```

❌ Wrong midpoint calculation

Always use

c=a+b/2

❌ Choosing wrong interval

Check sign carefully.

2-Minute Revision

1. Used to find roots.
2. Works for polynomial and transcendental equations.
3. Root lies where sign changes.
4. Condition:

$$f(a)f(b)<0$$

5. Divide interval repeatedly.
6. Midpoint formula:

$$c=a+b/2$$

7. Select new interval.
8. Repeat until accuracy achieved.
9. Simple and reliable.
10. Most asked RGPV numerical.

Memory Trick

BISECTION

BI = Break Interval

SECTION = Divide into Sections

Remember:

```
Find Interval
      ↓
Take Midpoint
      ↓
Check Sign
      ↓
```

```
Repeat
```

TOPIC 2: NEWTON-RAPHSON METHOD

What Is Newton-Raphson Method?

Newton-Raphson Method is a numerical technique used to find the root of an equation.

A root means the value of x that makes the equation equal to zero.

Unlike Bisection Method, this method uses the **derivative** of the equation.

It is one of the fastest methods for finding roots.

In RGPV exams, this is one of the most important topics of Unit 1.

Why Do We Need It?

Bisection Method is reliable but slow.

Engineers often need answers quickly.

Newton-Raphson Method reaches the root much faster.

Therefore it is widely used in:

- Engineering calculations
- Computer simulations
- Structural analysis
- Electrical systems

Real-Life Example

Imagine you are standing on a hill and want to reach the lowest point.

Instead of taking small random steps, you look at the slope and move directly towards the bottom.

Newton-Raphson Method also uses the slope (derivative) to move quickly toward the root.

Basic Idea

Suppose equation is:

$f(x)=0$

Choose an initial guess:

x_0

Then improve the guess repeatedly.

Every new value gets closer to the actual root.

Most Important Formula

This formula comes almost every year.

xn+1=xn−f(xn)/f′(xn)

Where:

x_n = current approximation

x_{n+1} = next approximation

$f(x_n)$ = function value

$f'(x_n)$ = derivative value

Understanding the Formula

Suppose:

Current guess = 2

Function value = 3

Slope = 6

Then:

New guess

= Old Guess − Error/Slope

This automatically moves us closer to root.

Step-by-Step Procedure

Step 1

Write equation.

Example:

$x^3-x-1=0$

Step 2

Find derivative.

$f'(x)=3x^2-1$

Step 3

Choose initial value.

$x_0 = 1.5$

Step 4

Substitute into formula.

Find x_1.

Step 5

Repeat.

Find x_2.

Find x_3.

Continue until answer stabilizes.

Diagram

```
Actual Root
     ●
    /
   /
  /
 /
● Initial Guess

Each iteration moves closer
to actual root.
```

Advantages

✅ Very fast

✅ High accuracy

✅ Less iterations

✅ Popular engineering method

✅ Useful for complex equations

Disadvantages

❌ Requires derivative

❌ Wrong initial guess may fail

❌ May diverge

❌ Not always stable

❌ Difficult for beginners

Common Mistakes

❌ Forget derivative

❌ Wrong derivative

❌ Calculator mistakes

❌ Choosing bad initial value

❌ Stopping too early

Memory Trick

NRS

N → Near Guess

R → Reduce Error

S → Solve Fast

Remember:

Newton = Fast Root Finder

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Repeated Questions:

- Explain Newton-Raphson Method.
- Solve equation using Newton-Raphson.
- Compare with Bisection Method.

2-Minute Revision

1. Fast root-finding method.
2. Uses derivative.
3. Requires initial guess.
4. Formula most important.
5. High accuracy.
6. Fewer iterations.
7. Exam favorite.

8. Used in engineering software.
9. Better than Bisection in speed.
10. May fail for wrong guess.

TOPIC 3: REGULA-FALSI METHOD

What Is It?

Regula-Falsi Method is also called:

False Position Method

It is another method to find roots of equations.

It combines ideas of:

- Bisection Method
- Secant Method

Why Do We Need It?

Bisection divides interval exactly in half.

Sometimes root is not near the midpoint.

Regula-Falsi uses a better estimate.

Therefore it often converges faster.

Real-Life Example

Imagine two friends standing at opposite ends of a road.

Instead of choosing the exact middle point, you draw a line between them and estimate where the target lies.

This estimated point becomes the next approximation.

Basic Idea

Take two points:

a and b

Join them using a straight line.

Where the line crosses x-axis becomes the next root estimate.

Most Important Formula

$x=af(b)-bf(a)/f(b)-f(a)$

Step-by-Step Procedure

Step 1

Choose interval.

Step 2

Check sign change.

Step 3

Apply Regula-Falsi formula.

Step 4

Calculate new root estimate.

Step 5

Replace interval.

Step 6

Repeat.

Diagram

```
f(x)

 |
 |\
 | \
 |  \
 |   \
 |    \
---------------------> x

Intersection point
gives root estimate
```

Advantages

✅ Faster than Bisection

✅ More accurate

✅ Easy implementation

✅ Stable method

✅ Widely used

Disadvantages

❌ Slower than Newton-Raphson

❌ More calculations

❌ Requires interval

❌ Iterative process

❌ Can become slow for some functions

Memory Trick

Regula-Falsi

= False Position

Remember:

Not midpoint, better position.

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Frequently Asked:

- Explain Regula-Falsi Method.

- Numerical based on Regula-Falsi.
- Compare with Bisection.

2-Minute Revision

1. False Position Method.
2. Root-finding technique.
3. Uses interval.
4. Uses straight-line approximation.
5. Faster than Bisection.
6. Formula very important.
7. Repeated RGPV question.
8. Good accuracy.
9. Numerical expected.
10. Easy to learn.

REGULA-FALSI METHOD (FALSE POSITION METHOD)

What Is Regula-Falsi Method?

Regula-Falsi Method is a numerical method used to find the root of an equation.

A root means the value of x for which:

$f(x)=0$

It is also called the **False Position Method** because it assumes the root lies at a calculated position between two points.

This method is faster than Bisection Method and is one of the most important topics in RGPV Mathematics-III.

Why Do We Need It?

Many equations cannot be solved directly.

Example:

x^3-x-1=0

There is no simple algebraic method to find the exact root.

Therefore we use numerical methods.

Regula-Falsi provides a better approximation than Bisection Method.

Real-Life Example

Imagine two friends standing at opposite ends of a road.

You want to estimate where a hidden treasure lies.

Instead of checking the middle point, you draw a straight line between the two friends and estimate a more accurate location.

That estimated location is called the **False Position**.

Basic Idea

Suppose:

f(a) is negative

f(b) is positive

Then root must lie between a and b.

Instead of taking the midpoint like Bisection Method, we join the points with a straight line.

Where that line cuts the x-axis becomes the next approximation.

Mathematical Formula

The most important formula of Regula-Falsi Method:

$x=af(b)-bf(a)/f(b)-f(a)$

Where:

- a = first point
- b = second point
- f(a) = function value at a
- f(b) = function value at b
- x = estimated root

⭐⭐⭐⭐⭐ Very Important Formula

Step-by-Step Procedure

Step 1

Given equation:

$f(x)=0$

Step 2

Choose two values:

a and b

such that:

$f(a) \times f(b) < 0$

This confirms root exists between them.

Step 3

Apply Regula-Falsi Formula

Calculate x.

Step 4

Find f(x).

Step 5

Check sign.

If:

$f(a) \times f(x) < 0$

New interval becomes:

[a,x]

Otherwise:

[x,b]

Step 6

Repeat process until desired accuracy is achieved.

Graphical Understanding

```
f(x)

 ^
 |
 |\
 | \
 |  \
 |   \
 |    \
 |     \
 +------\----------> x
        Root
```

The straight line intersects the x-axis.

That intersection point gives the next approximation.

Complete Numerical Example

Find root of:

x^3-x-1=0

Step 1

Take:

a = 1

b = 2

Step 2

Calculate:

f(1)

$= 1^3 - 1 - 1$

= -1

f(2)

$= 8 - 2 - 1$

= 5

Since:

$(-1)(5) < 0$

Root exists.

Step 3

Apply Formula

$x = 1(5) - 2(-1)/5 - (-1)$

$x = 7/6 = 1.1667$

Step 4

Calculate:

f(1.1667)

≈ -0.578

Still negative.

Therefore root lies between:

1.1667 and 2

Repeat process.

After several iterations:

Root ≈ 1.3247

Why Is It Faster Than Bisection?

Bisection always takes midpoint.

Example:

1 and 2

Midpoint = 1.5

But root may not be near 1.5.

Regula-Falsi intelligently estimates a better position.

Therefore fewer iterations are required.

Difference Between Bisection and Regula-Falsi

Bisection Method	Regula-Falsi Method
Uses midpoint	Uses straight-line approximation
Simpler	Slightly complex
Slower	Faster
Guaranteed convergence	Usually converges faster

Bisection Method	Regula-Falsi Method
Equal division	Unequal division

Advantages

1. Better Accuracy

Provides better approximation.

2. Faster

Usually faster than Bisection Method.

3. Easy Implementation

Formula is simple.

4. No Derivative Needed

Unlike Newton-Raphson.

5. Stable Method

Generally reliable.

6. Good for Engineering Problems

Used in numerical computations.

Disadvantages

1. More Calculations

Compared to Bisection.

2. Requires Initial Interval

Need sign change.

3. Iterative Method

Requires repeated calculations.

4. Not Fastest

Newton-Raphson is faster.

5. Can Converge Slowly

For some equations.

Important Keywords for Exam

- Root Finding
- False Position
- Numerical Method
- Approximation
- Convergence
- Sign Change
- Iteration
- Straight Line Approximation

Frequently Asked RGPV Questions

7 Marks

1. Explain Regula-Falsi Method.
2. Compare Bisection and Regula-Falsi.

3. Write algorithm of Regula-Falsi Method.

14 Marks

1. Explain Regula-Falsi Method with numerical example.
2. Derive Regula-Falsi Formula.
3. Solve equation using Regula-Falsi Method.

Common Mistakes Students Make

❌ Forget checking sign change.

❌ Wrong substitution in formula.

❌ Arithmetic mistakes.

❌ Wrong interval selection.

❌ Stopping iterations too early.

Memory Trick

"RF = Real Faster"

R → Regula

F → Faster than Bisection

Remember:

Bisection = Middle Point

Regula-Falsi = Better Point

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Repeated in RGPV Exams:

- Explain Regula-Falsi Method.
- Numerical using Regula-Falsi.
- Comparison with Bisection Method.
- Derivation of Regula-Falsi Formula.

30-Second Revision

1. Regula-Falsi = False Position Method.
2. Used to find roots.
3. Root means $f(x)=0$.
4. Requires sign change.
5. Uses straight-line approximation.
6. Formula is most important.
7. Faster than Bisection.
8. No derivative required.
9. Frequently asked in RGPV.
10. Root obtained after repeated iterations.

FINITE DIFFERENCES

What Is Finite Difference?

Finite Difference means **difference between two consecutive values** of a function.

In simple words, if we know values of y at different points, then the difference between these values is called finite difference.

Example:

x	**y**
1	5
2	8
3	12

Difference:

8 − 5 = 3

12 − 8 = 4

These are finite differences.

Finite differences are used in **Interpolation**, **Numerical Analysis**, and **Engineering Calculations**.

Why Do We Need Finite Differences?

Suppose we know:

Year	**Population**
2020	1000
2021	1200
2022	1450

We want to estimate population for 2021.5.

Finite differences help us estimate unknown values.

This process is called **Interpolation**.

Real-Life Example

Imagine your bank balance:

Day 1 = ₹1000

Day 2 = ₹1200

Day 3 = ₹1500

Difference:

1200 −1000 = 200

1500 −1200 = 300

These changes are finite differences.

Basic Idea

Finite Difference simply measures:

How much one value changes compared to another.

Mathematically:

Difference = New Value − Old Value

Types of Finite Differences

There are mainly three types:

1. Forward Difference (Δ)

2. Backward Difference (∇)

3. Central Difference (δ)

---

1. Forward Difference

Definition

Difference between a value and its next value.

Formula:

$\Delta y = y_{i+1} - y_i$

Example

x	y
1	10
2	15
3	22

Forward Difference:

15−10 = 5

22−15 = 7

Table

```
y      10    15    22

Δy           5     7
```

---

2. Backward Difference

Definition

Difference between current value and previous value.

Formula:

$\nabla y = y_i - y_{i-1}$

Example

x	y
1	10
2	15
3	22

Backward Difference:

15−10 = 5

22−15 = 7

Table

```
y       10    15    22

∇y             5     7
```

---

3. Central Difference

Definition

Difference calculated around a central value.

Formula:

$\delta y = y_{i+1/2} - y_{i-1/2}$

Used in advanced numerical calculations.

Less important for basic RGPV questions.

Difference Table

Consider:

x	y
0	1
1	8
2	27
3	64

Step 1

First Difference

```
1      8      27      64
   7      19      37
```

Step 2

Second Difference

1 8 27 64

7 19 37

12 18

Step 3

Third Difference

1 8 27 64

7 19 37

12 18

6

Orders of Difference

First Difference

Δy

Second Difference

Δ^2y

Third Difference

$\Delta^3 y$

Fourth Difference

$\Delta^4 y$

Operators in Finite Difference

Very Important Theory Question

⭐⭐⭐⭐⭐

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1. Forward Difference Operator (Δ)

Formula:

$\Delta y = y_{i+1} - y_i$

Purpose:

Find next difference.

---

2. Backward Difference Operator (∇)

Formula:

$\nabla y = y_i - y_{i-1}$

Purpose:

Find previous difference.

---

3. Shift Operator (E)

Formula:

$Ey=y(x+h)$

Purpose:

Shift function by h units.

---

4. Differential Operator (D)

Formula:

$Dy=dy/dx$

Purpose:

Differentiation.

Relation Between Operators

Most Important Theory Question

⭐⭐⭐⭐⭐

Relation 1

$\Delta=E-1$

Relation 2

$\nabla=1-E^{-1}$

Relation 3

$E=e^{hD}$

These relations are frequently asked in RGPV exams.

Advantages

1. Easy Calculations

Simple subtraction.

2. Used in Interpolation

Foundation of interpolation formulas.

3. Useful in Numerical Analysis

Engineering calculations become easier.

4. Helps Approximate Unknown Values

Useful when exact value unavailable.

5. Computer Friendly

Easy for software implementation.

Disadvantages

1. Approximate Results

Not exact.

2. Error Accumulation

Errors may increase.

3. Large Tables Required

Need sufficient data.

4. Complex for Higher Orders

Higher differences become difficult.

5. Not Suitable for Random Data

Works best for organized data.

Important Keywords for Exam

- Finite Difference
- Forward Difference
- Backward Difference
- Central Difference
- Difference Table
- Shift Operator
- Differential Operator
- Interpolation
- Numerical Analysis

Frequently Asked RGPV Questions

7 Marks

1. Define Finite Difference.
2. Explain Forward Difference Operator.
3. Explain Backward Difference Operator.
4. Discuss Difference Table.

14 Marks

1. Explain finite differences and their types.
2. Derive relation between operators.
3. Explain difference operators with examples.
4. Construct finite difference table.

Common Mistakes Students Make

❌ Wrong subtraction.

❌ Mixing forward and backward difference.

❌ Wrong placement in difference table.

❌ Forgetting operator symbols.

❌ Not learning operator relations.

Memory Trick

Remember:

FBC Rule

F → Forward Difference (Δ)

B → Backward Difference (∇)

C → Central Difference (δ)

Operator Trick

"DEN"

D → Differential

E → Shift

N → Nabla (Backward)

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

- Forward Difference Operator
- Backward Difference Operator
- Relation Between Operators

⭐⭐⭐⭐ High Probability

- Difference Table
- Higher Order Differences

⭐⭐⭐ Medium Probability

- Central Difference

30-Second Revision

1. Finite Difference = Difference between successive values.
2. Used in interpolation.
3. Forward Difference = Δ.
4. Backward Difference = ∇.
5. Central Difference = δ.
6. Difference table is important.
7. Higher differences are obtained repeatedly.
8. Shift Operator = E.

9. Differential Operator = D.
10. Relation: $\Delta = E - 1$.
11. Relation: $E = e^{(hD)}$.
12. Frequently asked RGPV topic.
13. Basis of Newton Forward Formula.
14. Basis of Newton Backward Formula.
15. Must study before interpolation. ⭐⭐⭐⭐⭐

RELATION BETWEEN OPERATORS (Finite Difference Operators)

⭐⭐⭐⭐⭐ **Most Important Theory Question for RGPV**

This topic is asked very frequently in Mathematics-III exams.

Many students memorize formulas but do not understand them. Let's learn it from zero level.

What Is an Operator?

An operator is a mathematical symbol that performs a specific operation on a function.

Think of operators like buttons on a calculator.

Example:

-
    - performs addition
- − performs subtraction
- × performs multiplication

Similarly:

- Δ finds Forward Difference
- ∇ finds Backward Difference

- E shifts a function
- D differentiates a function

Why Do We Need Relations Between Operators?

Different interpolation formulas use different operators.

Instead of solving everything separately, mathematicians developed relationships between operators.

These relations make calculations easier.

Important Operators

Symbol	Name
Δ	Forward Difference Operator
∇	Backward Difference Operator
E	Shift Operator
D	Differential Operator

---

1. Forward Difference Operator (Δ)

Definition

Forward difference means:

Next Value − Current Value

Formula:

$\Delta y = y(x+h) - y(x)$

Example

Suppose:

x	y
1	10
2	15

Forward Difference

15 − 10 = 5

---

2. Shift Operator (E)

Definition

Shift Operator moves function by h units.

Formula:

$Ey = y(x+h)$

Meaning:

Replace x with (x+h)

Example

Suppose

$y = x^2$

Then

At $x = 2$

$y = 4$

If $h = 1$

Then

$E(y)$

$= y(2+1)$

$= y(3)$

$= 9$

Relation 1 : $\Delta = E - 1$

⭐⭐⭐⭐⭐ Very Important

Derivation

We know:

Forward Difference

$\Delta y = y(x+h) - y(x)$

Using Shift Operator:

$Ey = y(x+h)$

Substitute:

$\Delta y = Ey - y$

Taking y common:

$\Delta y=(E-1)y$

Therefore:

Final Relation

$\Delta=E-1$

Memory Trick

Forward Difference

= Shift − Original

Therefore:

$\Delta = E - 1$

---

3. Backward Difference Operator (∇)

Definition

Backward Difference means:

Current Value − Previous Value

Formula:

$\nabla y=y(x)-y(x-h)$

Relation 2 : $\nabla = 1 - E^{-1}$

⭐⭐⭐⭐⭐ Important

Derivation

We know:

$\nabla y = y(x) - y(x-h)$

Since:

$E^{-1} y = y(x-h)$

Substitute:

$\nabla y = y - E^{-1} y$

Taking common:

$\nabla y = (1 - E^{-1}) y$

Hence:

Final Relation

$\nabla = 1 - E^{-1}$

Memory Trick

Backward

= Current − Previous

Therefore:

$\nabla = 1 - E^{-1}$

---

4. Differential Operator (D)

Definition

Differential Operator means differentiation.

Formula:

$Dy=dy/dx$

Example

Suppose

$y = x^2$

Then

$Dy=2x$

Relation 3 : E = e^(hD)

⭐⭐⭐⭐⭐ Most Important

Meaning

Shift Operator and Differentiation Operator are related.

Formula:

$E=e^{hD}$

Where:

h = interval

D = differential operator

Exam Point

You are not usually required to derive this in detail in RGPV.

Just remember formula.

Other Important Relations

Relation 4

$E=1+\Delta$

Obtained from:

$\Delta = E - 1$

Relation 5

$E=(1-\nabla)^{-1}$

Relation 6

$\Delta=E\nabla$

Summary Table

Relation	Importance
$\Delta = E - 1$	⭐⭐⭐⭐⭐
$E = 1 + \Delta$	⭐⭐⭐⭐⭐
$\nabla = 1 - E^{-1}$	⭐⭐⭐⭐⭐
$E = e^{hD}$	⭐⭐⭐⭐⭐
$\Delta = E\nabla$	⭐⭐⭐⭐

Important Keywords

- Forward Difference
- Backward Difference
- Shift Operator
- Differential Operator
- Finite Difference
- Interpolation
- Numerical Methods

Frequently Asked RGPV Questions

7 Marks

1. Define Shift Operator.
2. Explain Forward Difference Operator.
3. Explain Backward Difference Operator.
4. Derive $\Delta = E - 1$.

14 Marks

1. Derive relations among finite difference operators.
2. Explain Forward, Backward and Shift Operators.
3. Derive $\Delta = E - 1$ and $\nabla = 1 - E^{-1}$.

Common Mistakes Students Make

❌ Writing $\Delta = 1 - E$

Correct:

$\Delta = E - 1$

❌ Writing $\nabla = E - 1$

Correct:

$\nabla = 1 - E^{-1}$

❌ Forgetting inverse in backward relation

Always remember:

Backward = E^{-1}

2-Minute Revision

1. Δ = Forward Difference.
2. ∇ = Backward Difference.
3. E = Shift Operator.
4. D = Differential Operator.
5. $\Delta = E - 1$.
6. $E = 1 + \Delta$.
7. $\nabla = 1 - E^{-1}$.
8. E = e^(hD).
9. Very important theory question.
10. Frequently asked in RGPV.
11. Basis of interpolation formulas.
12. Must memorize all relations.

Super Memory Trick

"FED"

F → Forward → $\Delta = E - 1$

E → Shift → $E = e^{(hD)}$

D → Difference Relations

Memorize these three:

$\Delta=E-1$

$\nabla=1-E^{-1}$

$E=e^{hD}$

These alone can fetch marks in almost every RGPV Unit-1 theory question on operators. ⭐⭐⭐⭐⭐.

INTERPOLATION USING NEWTON'S FORWARD AND BACKWARD DIFFERENCE FORMULAE

⭐⭐⭐⭐⭐ **Most Important Topic of Module 1**

This topic is asked almost every year in RGPV.

Usually:

- 7 Marks Numerical
- 14 Marks Derivation + Numerical

So understand it properly.

First Understand: What Is Interpolation?

What Is It?

Interpolation means finding an unknown value between known values.

Suppose:

Year	Population
2020	1000
2021	1200
2022	1450

You want population in 2021.5

Since 2021.5 lies between known values, we estimate it.

This estimation process is called **Interpolation**.

Real Life Example

Suppose:

At 9 AM = 20°C

At 10 AM = 24°C

What was temperature at 9:30 AM?

You estimate between known values.

This is interpolation.

Why Do We Need Interpolation?

Used in:

- Engineering calculations
- Weather prediction
- Population studies
- Machine learning
- Signal processing

Whenever exact value is unavailable, interpolation helps.

Types of Newton Interpolation

There are two important formulas:

1. Newton Forward Interpolation

Used when required value is near beginning.

2. Newton Backward Interpolation

Used when required value is near end.

NEWTON FORWARD INTERPOLATION

What Is Newton Forward Interpolation?

Used when unknown value lies near the first entry of the table.

Example:

x	y
10	100
20	400
30	900

x	y
40	1600

Need value near x=12

Since 12 is close to first value (10)

Use Newton Forward Formula.

Memory Trick

Forward = Front

F → Front

If value is near beginning

Use Newton Forward.

Formula

⭐⭐⭐⭐⭐ Most Important

$y=y0+p\Delta y0+p(p-1)/2!\Delta 2y0+p(p-1)(p-2)/3!\Delta 3y0+\cdots$

Meaning of Symbols

y_0

First value of y

Δy_0

First forward difference

$\Delta^2 y_0$

Second forward difference

$\Delta^3 y_0$

Third forward difference

---

p

Most Important

p=x−x0hp=\frac{x-x_0}{h}p=hx−x0

Where:

x = required value

x_0 = first value

h = interval

Step-by-Step Procedure

Step 1

Make difference table.

Step 2

Calculate p.

Step 3

Apply formula.

Step 4

Substitute values.

Step 5

Calculate answer.

Difference Table Example

x	y	Δy	$\Delta^2 y$
0	1	7	12
1	8	19	
2	27		

Diagram

```
Beginning
↓
10   20   30   40

Need x=12

Use Newton Forward
```

Advantages

✅ Easy

✅ Accurate

✅ Uses forward differences

✅ Good for beginning values

✅ Frequently asked

Disadvantages

❌ Not suitable near end

❌ Requires equal interval

❌ Long calculations

❌ Error accumulation

❌ Large tables needed

Frequently Asked Questions

1. Derive Newton Forward Formula.
2. Explain Newton Forward Interpolation.
3. Solve numerical using Newton Forward.

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Almost every year.

2-Minute Revision

1. Used near beginning.
2. Uses forward difference.
3. Equal interval required.
4. Calculate p.
5. Apply formula.
6. Very important RGPV topic.

NEWTON BACKWARD INTERPOLATION

What Is Newton Backward Interpolation?

Used when required value lies near the last entry.

Example:

x	y
10	100
20	400
30	900
40	1600

Need value near x=38

Since 38 is close to 40

Use Newton Backward Formula.

Memory Trick

Backward = Back

B → Back

If value is near end

Use Newton Backward.

Formula

⭐⭐⭐⭐⭐ Most Important

y=yn+p∇yn+p(p+1)/2!∇2yn+p(p+1)(p+2)/3!∇3yn+⋯

Meaning of Symbols

y_n

Last value

∇y_n

Backward difference

---

p

p=x−xn / h

Step-by-Step Procedure

Step 1

Construct backward difference table.

Step 2

Calculate p.

Step 3

Apply formula.

Step 4

Substitute values.

Step 5

Find answer.

Diagram

```
10   20   30   40
               ↑

Need x=38

Near last value

Use Newton Backward
```

Advantages

✅ Best near end

✅ Accurate

✅ Easy calculations

✅ Uses backward differences

✅ Frequently asked

Disadvantages

❌ Not suitable near beginning

❌ Equal interval needed

❌ Long calculations

❌ Error accumulation

❌ Large table required

Difference Between Forward and Backward

Newton Forward	Newton Backward
Beginning values	End values
Uses Δ	Uses ∇
$p=(x-x_0)/h$	$p=(x-x_n)/h$
Starts from first row	Starts from last row

Common Mistakes Students Make

❌ Using Forward when value is near end.

❌ Using Backward when value is near beginning.

❌ Wrong calculation of p.

❌ Wrong difference table.

❌ Forgetting factorial.

Important Keywords

- Interpolation
- Forward Difference
- Backward Difference
- Difference Table
- Equal Interval
- Approximation
- Numerical Methods

PYQ Analysis

Newton Forward

⭐⭐⭐⭐⭐ Very High Probability

Newton Backward

⭐⭐⭐⭐⭐ Very High Probability

Difference Between Them

⭐⭐⭐⭐

30-Second Revision

Newton Forward

- Used near beginning.
- Uses Δ.
- Formula starts from y_0.
- $p=(x-x_0)/h$.

Newton Backward

- Used near end.
- Uses ∇.
- Formula starts from y_n.
- $p=(x-x_n)/h$.

Golden Rule

📌 Near First Value → Forward

📌 Near Last Value → Backward

This single rule helps solve most interpolation numericals correctly in exams. ⭐⭐⭐⭐⭐

INTERPOLATION WITH UNEQUAL INTERVALS

⭐⭐⭐⭐⭐ Very Important RGPV Topic

When the spacing between x-values is not equal, Newton Forward and Newton Backward formulas cannot be used.

In such cases we use:

1. **Newton's Divided Difference Formula**

2. **Lagrange Interpolation Formula**

These are called **Interpolation with Unequal Intervals**.

First Understand Equal and Unequal Intervals

Equal Interval

x	y
1	2
2	4
3	8
4	16

Difference between x-values:

$2-1 = 1$

$3-2 = 1$

$4-3 = 1$

All intervals same.

So Newton Forward/Backward can be used.

Unequal Interval

x	y
1	2
3	10
7	50

x	y
10	100

Differences:

3−1 = 2

7−3 = 4

10−7 = 3

Intervals are different.

Therefore Newton Forward and Backward formulas fail.

Real-Life Example

Suppose temperature is recorded at:

Time	Temp
9 AM	20°C
11 AM	28°C
2 PM	36°C

Measurements are not taken at equal intervals.

To estimate temperature at 12 PM, we use interpolation for unequal intervals.

Methods Used

1. Newton's Divided Difference Formula

2. Lagrange Interpolation Formula

PART 1 : NEWTON'S DIVIDED DIFFERENCE FORMULA

What Is Newton's Divided Difference Formula?

It is an interpolation method used when x-values are not equally spaced.

It is one of the most important numerical methods.

Why Do We Need It?

Newton Forward Formula requires equal intervals.

When intervals become unequal, divided differences are used.

Basic Idea

Instead of ordinary differences:

Δy

We calculate:

Divided Differences

These differences are divided by corresponding x differences.

First Divided Difference

Formula:

$f[x_0,x_1]=f(x_1)-f(x_0) / x_1-x_0$

Second Divided Difference

Formula:

$f[x_0,x_1,x_2]=f[x_1,x_2]-f[x_0,x_1] / x_2-x_0$

General Formula

$P(x)=f(x_0)+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+\cdots$

Step-by-Step Procedure

Step 1

Create divided difference table.

Step 2

Calculate first divided differences.

Step 3

Calculate second divided differences.

Step 4

Apply Newton Divided Difference Formula.

Step 5

Substitute required value.

Step 6

Calculate answer.

Diagram

```
x0 ---- x1 -------- x2 --- x3

Unequal Distances

Use Divided Difference
```

Advantages

✅ Works for unequal intervals

✅ Accurate

✅ Easy table construction

✅ Widely used

✅ Suitable for engineering data

Disadvantages

❌ Long calculations

❌ More arithmetic work

❌ Error accumulation

❌ Difficult for beginners

❌ Large tables required

Important Keywords

- Divided Difference
- Unequal Interval
- Interpolation
- Numerical Analysis
- Approximation

Memory Trick

NDD

Newton

Divided

Difference

Remember:

Unequal Interval → Divided Difference

PYQ Analysis

⭐⭐⭐⭐⭐ Very High Probability

Frequently Asked:

1. Explain Divided Difference Method.
2. Solve numerical using Divided Difference.
3. Construct Divided Difference Table.

PART 2 : LAGRANGE INTERPOLATION FORMULA

What Is Lagrange Interpolation?

Lagrange Interpolation is another method used for unequal intervals.

Unlike Newton's method, no difference table is needed.

Direct formula is used.

Why Do We Need It?

Sometimes creating divided difference tables becomes lengthy.

Lagrange Formula directly gives interpolation polynomial.

Real-Life Example

Suppose we know:

Marks	Grade
40	Pass
60	First
80	Distinction

Need estimate at 70.

Lagrange formula can be used.

Basic Idea

Construct polynomial passing through all known points.

Then use that polynomial to find unknown values.

Formula

For three points:

P(x)=y0L0+y1L1+y2L2

Where

First Basis Function

L0=(x−x1)(x−x2)/(x0−x1)(x0−x2)

Second Basis Function

L1=(x−x0)(x−x2)/(x1−x0)(x1−x2)

Third Basis Function

$L_2=(x-x_0)(x-x_1)/(x_2-x_0)(x_2-x_1)$

Step-by-Step Procedure

Step 1

Write all known points.

Step 2

Calculate basis functions.

Step 3

Substitute values.

Step 4

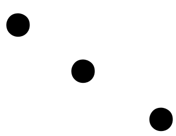
Find interpolation polynomial.

Step 5

Calculate required value.

Diagram

Known Points



```
Curve passes through
all points

Lagrange Formula
```

Advantages

✅ No difference table

✅ Works for unequal intervals

✅ Accurate

✅ Direct formula

✅ Frequently asked in exams

Disadvantages

❌ Long calculations

❌ Difficult algebra

❌ Not suitable for many points

❌ Time consuming

❌ Large expressions

Difference Between Newton Divided Difference and Lagrange

Newton Divided Difference	Lagrange
Uses table	No table
Easier for many points	Difficult for many points
Stepwise calculation	Direct formula
Frequently used	Frequently used
Efficient updating	Not efficient

Common Mistakes Students Make

❌ Using Forward Formula for unequal intervals.

❌ Wrong divided difference calculation.

❌ Wrong basis functions.

❌ Arithmetic mistakes.

❌ Ignoring denominator signs.

Frequently Asked RGPV Questions

7 Marks

1. Explain Divided Difference Method.
2. Explain Lagrange Formula.
3. Difference between Newton and Lagrange methods.

14 Marks

1. Derive Newton Divided Difference Formula.
2. Solve numerical using Divided Difference.
3. Derive Lagrange Interpolation Formula.
4. Solve numerical using Lagrange Formula.

2-Minute Revision

Equal Interval

✅ Newton Forward

✅ Newton Backward

Unequal Interval

✅ Newton Divided Difference

✅ Lagrange Formula

Golden Exam Rule

```
Equal Interval
       ↓
Forward / Backward

Unequal Interval
       ↓
Divided Difference
or
Lagrange Formula
```

NEWTON'S DIVIDED DIFFERENCE AND LAGRANGE'S FORMULAE

⭐⭐⭐⭐⭐ **RGPV Exam Favourite Topic**

(Almost every year 7–14 marks question)

This topic comes under **Interpolation with Unequal Intervals**.

---

1. NEWTON'S DIVIDED DIFFERENCE FORMULA

What Is It?

Newton's Divided Difference Formula is used to find unknown values when the x-values are not equally spaced.

If the interval between x-values is unequal, Newton Forward and Backward formulas cannot be used.

Then we use Divided Difference Method.

Why Do We Need It?

Suppose data is:

x	y
1	5
3	15
7	45
10	80

Here intervals are:

$3-1 = 2$

$7-3 = 4$

$10-7 = 3$

Intervals are unequal.

Hence Newton Forward/Backward fails.

Real-Life Example

Suppose temperature is recorded at:

9 AM → 20°C

11 AM → 25°C

2 PM → 35°C

Time intervals are unequal.

To estimate temperature at 12 PM, we use Divided Difference.

Basic Idea

Instead of ordinary differences (Δy),

we divide differences by differences of x.

Thus:

```
Difference
-----------
x Difference
```

This is called Divided Difference.

First Divided Difference

Formula

$f[x_0,x_1]=f(x_1)-f(x_0) / x_1-x_0$

Second Divided Difference

Formula

$f[x_0,x_1,x_2]=f[x_1,x_2]-f[x_0,x_1] / x_2-x_0$

Newton Divided Difference Formula

⭐⭐⭐⭐⭐ Most Important Formula

$P(x)=f(x_0)+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+\cdots$

Step-by-Step Procedure

Step 1

Create Divided Difference Table.

Step 2

Calculate First Divided Difference.

Step 3

Calculate Second Divided Difference.

Step 4

Apply Formula.

Step 5

Substitute required x value.

Step 6

Find answer.

Diagram

```
x0 ----- x1 -------- x2 ---- x3

Unequal Distances

Use Divided Difference
```

Advantages

✅ Works for unequal intervals

✅ Accurate

✅ Easy to update table

✅ Suitable for engineering data

✅ Frequently asked in exams

Disadvantages

❌ Lengthy calculations

❌ Arithmetic mistakes possible

❌ Difficult for beginners

❌ Large tables required

❌ Time consuming

Applications

✅ Numerical Analysis

✅ Weather Forecasting

✅ Engineering Measurements

✅ Scientific Calculations

✅ Data Approximation

Important Keywords

- Divided Difference
- Unequal Interval
- Interpolation
- Approximation
- Numerical Method

Memory Trick

UDI

Unequal

Difference

Interpolation

Whenever interval is unequal → Divided Difference

Frequently Asked RGPV Questions

1. Derive Newton's Divided Difference Formula.
2. Construct Divided Difference Table.
3. Solve numerical using Divided Difference.
4. Explain interpolation with unequal intervals.

Exam Focus

⭐ PYQ Frequency → Very High

⭐⭐⭐⭐⭐ Probability

Most Important Diagram:

Divided Difference Table

Most Important Formula:

$f[x0,x1]=f(x1)-f(x0) / x1-x0$

LAGRANGE INTERPOLATION FORMULA

What Is It?

Lagrange Formula is another interpolation technique for unequal intervals.

Unlike Newton's method,

No difference table is required.

Direct formula is used.

Why Do We Need It?

Sometimes constructing difference tables becomes lengthy.

Lagrange gives direct interpolation.

Real-Life Example

Suppose:

Age	Height
10	130
12	145
15	165

Need height at age 13.

Lagrange Formula can estimate it.

Basic Idea

Construct a polynomial that passes through all known points.

Then use that polynomial to estimate unknown values.

Formula

For three points

$P(x)=y_0L_0+y_1L_1+y_2L_2$

Where

First Basis Function

$L_0=(x-x_1)(x-x_2)/(x_0-x_1)(x_0-x_2)$

Second Basis Function

$L_1=(x-x_0)(x-x_2)/(x_1-x_0)(x_1-x_2)$

Third Basis Function

$L_2=(x-x_0)(x-x_1)/(x_2-x_0)(x_2-x_1)$

Step-by-Step Procedure

Step 1

Write all known points.

Step 2

Find basis functions.

Step 3

Substitute values.

Step 4

Form interpolation polynomial.

Step 5

Calculate required value.

Diagram

```
Known Points

●

      ●

             ●

Polynomial passes
through all points
```

Advantages

✅ No difference table

✅ Direct method

✅ Accurate

✅ Works for unequal intervals

✅ Easy for small data

Disadvantages

❌ Long algebraic calculations

❌ Difficult for many points

❌ Time consuming

❌ Not efficient for updates

❌ Complex expressions

Applications

✅ Engineering Calculations

✅ Scientific Data Analysis

✅ Computer Graphics

✅ Machine Learning

✅ Signal Processing

Important Keywords

- Lagrange Polynomial
- Basis Function
- Interpolation
- Approximation
- Unequal Interval

Memory Trick

LAG

L → Large Formula

A → Any Interval

G → Gives Polynomial

Frequently Asked RGPV Questions

1. Derive Lagrange Interpolation Formula.
2. Explain Lagrange Method.
3. Solve numerical using Lagrange Formula.
4. Compare Newton and Lagrange Methods.

Difference Between Newton and Lagrange

Newton Divided Difference	Lagrange
Uses Difference Table	No Table
Stepwise Method	Direct Method
Easier for Large Data	Difficult for Large Data
Easy Updating	Not Easy Updating
More Popular	Simpler Concept

PYQ ANALYSIS

Newton Divided Difference

⭐⭐⭐⭐⭐ Very High Probability

Repeatedly Asked

Lagrange Formula

⭐⭐⭐⭐⭐ Very High Probability

Repeatedly Asked

Super Revision Sheet

```
Equal Interval
↓
Newton Forward
Newton Backward

Unequal Interval
↓
Newton Divided Difference
Lagrange Formula
```

Must Remember

1. Divided Difference → Unequal Interval
2. Lagrange → Unequal Interval
3. Newton Forward → Equal Interval
4. Newton Backward → Equal Interval

Most Important Questions

⭐⭐⭐⭐⭐ Derive Newton Divided Difference Formula

⭐⭐⭐⭐⭐ Derive Lagrange Formula

⭐⭐⭐⭐⭐ Numerical on Divided Difference

⭐⭐⭐⭐⭐ Numerical on Lagrange Formula

These two topics alone frequently contribute **10–20 marks** in RGPV Mathematics-III Module-1 examinations.