Engineering Mathematics-II (BT-202) Module-5 Notes Vector Calculus

MODULE-5: Vector Calculus

Topics Covered
- Differentiation of Vectors
- Scalar and Vector Point Function
- Gradient
- Geometrical Meaning of Gradient
- Directional Derivative
- Divergence and Curl
- Line Integral
- Surface Integral
- Volume Integral
- Gauss Divergence Theorem
- Stokes Theorem
- Green’s Theorem

1. VECTOR FUNCTION

A vector function is a function whose value is a vector.

Example:

$r(t)=x(t)i+y(t)j+z(t)k$

Where x(t), y(t), z(t) are scalar functions.

2. DIFFERENTIATION OF VECTOR FUNCTION

If:

$r(t)=x(t)i+y(t)j+z(t)k$

Then:

$dr/dt = (dx/dt)i + (dy/dt)j + (dz/dt)k$

Applications:
- Velocity vector
- Acceleration vector

3. SCALAR POINT FUNCTION

A scalar point function assigns scalar value to each point in space.

Example:

$\varphi(x,y,z)=x^2+y^2+z^2$

4. VECTOR POINT FUNCTION

A vector point function assigns vector to each point in space.

Example:

$F(x,y,z)=xi+yj+zk$

5. GRADIENT

Gradient of scalar function φ is denoted by grad φ or ∇φ.

Formula:

$\nabla\varphi = (\partial\varphi/\partial x)i + (\partial\varphi/\partial y)j + (\partial\varphi/\partial z)k$

Geometrical Meaning:
Gradient gives direction of maximum increase of scalar function.

6. DIRECTIONAL DERIVATIVE

Rate of change of scalar function in a particular direction.

Formula:

Directional Derivative = ∇φ · a■

Where a■ is unit vector in required direction.

7. DIVERGENCE

Divergence measures outward flow of vector field.

Formula:

$\text{div } F = \nabla\cdot F$

If:

F=F■i+F■j+F■k

Then:

∇·F = ∂F■/∂x + ∂F■/∂y + ∂F■/∂z

8. CURL

Curl measures rotation of vector field.

Formula:

$\text{curl } F = \nabla\times F$

Expanded form:

∇×F = | i j k |
| ∂/∂x ∂/∂y ∂/∂z |
| F■ F■ F■ |

9. IRROTATIONAL VECTOR

If curl F = 0 then vector field is irrotational.

10. SOLENOIDAL VECTOR

If div F = 0 then vector field is solenoidal.

11. LINE INTEGRAL

Integral of vector function along a curve is called line integral.

Formula:

∫C F·dr

Applications:
• Work done by force field

12. SURFACE INTEGRAL

Integral over surface S.

Formula:

■S F·n■ dS

13. VOLUME INTEGRAL

Integral over volume V.

Formula:

■V f(x,y,z)dV

14. GAUSS DIVERGENCE THEOREM

States that surface integral over closed surface equals volume integral of divergence.

Formula:

■S F·n■ dS = ■V (∇·F)dV

Applications:
• Fluid flow problems
• Electromagnetic theory

15. STOKES THEOREM

Relates line integral around closed curve to surface integral of curl.

Formula:

■C F·dr = ■S (∇×F)·n■ dS

Applications:
• Circulation problems
• Fluid mechanics

16. GREEN’S THEOREM

Relates line integral around simple closed curve to double integral over enclosed region.

Formula:

■C (Pdx + Qdy) = ■R [(∂Q/∂x)-(∂P/∂y)] dA

Applications:
• Area calculation
• Plane vector fields

17. IMPORTANT RESULTS

1. Gradient is normal to level surface.

2. Divergence measures source strength.

3. Curl measures rotational effect.

4. Conservative field has zero curl.

MOST IMPORTANT 14 MARK QUESTIONS

1. Find gradient and directional derivative of scalar functions.

2. Explain geometrical meaning of gradient.

3. Find divergence and curl of vector fields.

4. Verify vector field is solenoidal and irrotational.

5. Evaluate line integrals.

6. Evaluate surface and volume integrals.

7. State and prove Gauss Divergence theorem.

8. Verify Gauss Divergence theorem.

9. State and prove Stokes theorem.

10. Verify Stokes theorem.

11. State and prove Green’s theorem.

12. Evaluate line integrals using Green’s theorem.

IMPORTANT 7 MARK QUESTIONS

1. Define gradient.

2. Define divergence.

3. Define curl.

4. Define directional derivative.

5. Define line integral.

6. State Gauss Divergence theorem.

7. State Stokes theorem.

8. State Green’s theorem.

9. Define irrotational vector.

10. Define solenoidal vector.

EXAM TIPS

• Practice vector differentiation numericals daily.
• Learn formulas of gradient, divergence and curl carefully.
• Practice theorem verification problems.
• Revise vector identities regularly.
• Focus on Gauss, Stokes and Green theorem applications.