Engineering Mathematics-II (BT-202) Module-2 Notes

MODULE-2: Ordinary Differential Equations II

Topics Covered
- Second order linear differential equations with variable coefficients
- Method of variation of parameters
- Power series solutions
- Legendre polynomials
- Bessel functions of first kind and properties

1. Second Order Linear Differential Equation with Variable Coefficients

General form:
$y'' + P(x)y' + Q(x)y = R(x)$

If $R(x)=0$ then equation is homogeneous.
If $R(x)\neq 0$ then equation is non-homogeneous.

Important Methods:
- Cauchy-Euler Method
- Variation of Parameters
- Power Series Method

Cauchy-Euler Equation
$x^2y'' + axy' + by = 0$

Assume y = x^m
Substitute and obtain auxiliary equation.

2. Method of Variation of Parameters

Used for solving non-homogeneous differential equations.

Standard equation:
$y'' + P(x)y' + Q(x)y = R(x)$

Complementary function:
y_c = c■y■ + c■y■

Particular Integral:
y_p = u(x)y■ + v(x)y■

Formula:
u' = -y■R/W
v' = y■R/W

Where W is Wronskian:
W = y■y■' - y■y■'

General Solution:
y = y_c + y_p

3. Power Series Solution

Assume solution in series form:
y = a■ + a■x + a■x² + a■x³ + ...

Differentiate term by term and substitute into differential equation.

Find recurrence relation and coefficients.

Applications:
• Bessel Equation
• Legendre Equation

4. Legendre Differential Equation

$(1-x^2)y'' - 2xy' + n(n+1)y = 0$

Solutions are called Legendre Polynomials.

Legendre Polynomials:

P■(x)=1
P■(x)=x
P■(x)=1/2(3x^2-1)
P■(x)=1/2(5x^3-3x)

Properties of Legendre Polynomials

1. Orthogonality Property
2. Recurrence Relation
3. Rodrigues Formula

Rodrigues Formula:
P■(x)=1/(2■n!) d■/dx■[(x^2-1)■]

Recurrence Relation
(n+1)P■■■(x)=(2n+1)xP■(x)-nP■■■(x)

5. Bessel Differential Equation

$x^2y'' + xy' + (x^2-n^2)y = 0$

Solutions are called Bessel Functions.

Bessel Function of First Kind

J■(x)= Σ [(-1)^r / r!Γ(n+r+1)] (x/2)^(n+2r)

Important Bessel Functions

J■(x)=1 - x^2/4 + x■/64 - ...

J■(x)=x/2 - x^3/16 + x■/384 - ...

Properties of Bessel Functions

1. Recurrence Relations
2. Orthogonality
3. Generating Function

Important Relation
J■■(x)=(-1)■J■(x)

Most Important PYQ Questions

1. Solve Legendre differential equation.

2. Solve using variation of parameters.

3. Derive recurrence relation of Legendre polynomial.

4. Prove J■■(x)=(-1)■J■(x).

5. Solve Bessel differential equation.

6. Solve differential equations using power series method.

7. Derive Rodrigues Formula.

8. Find Bessel function of first kind.

Exam Tips

• Practice derivations carefully.
• Learn standard forms of Legendre and Bessel equations.
• Remember recurrence relations.
• Practice variation of parameters numericals.
• Revise formulas daily before exam.