

Engineering Mathematics Module-1 Notes Ordinary Differential Equations-I

MODULE-1 TOPICS

- Differential Equations of First Order and First Degree
- Leibnitz Linear Differential Equation
- Bernoulli's Differential Equation
- Exact Differential Equation
- Differential Equations of First Order and Higher Degree
- Higher Order Differential Equations with Constant Coefficients
- Homogeneous Linear Differential Equations
- Simultaneous Differential Equations

1. DIFFERENTIAL EQUATION

An equation containing derivatives of dependent variable with respect to independent variable is called differential equation.

Examples:

$$dy/dx + y = 0$$

$$d^2y/dx^2 + 3dy/dx + 2y = 0$$

Order of Differential Equation:

Highest order derivative present in equation.

Degree of Differential Equation:

Power of highest order derivative after removing radicals and fractions.

2. DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

General form:

$$dy/dx = f(x,y)$$

Methods of Solution:

- Variable separable method
- Homogeneous method
- Linear method
- Bernoulli's method
- Exact equation method

3. LINEAR DIFFERENTIAL EQUATION (LEIBNITZ FORM)

Standard form:

$$dy/dx + Py = Q$$

Where P and Q are functions of x.

Solution Steps:

1. Find Integrating Factor (I.F.)
2. Multiply equation by I.F.
3. Integrate both sides

Integrating Factor:

$$I.F. = e^{\int P dx}$$

General Solution:

$$y \times \text{I.F.} = \int Q \times \text{I.F.} \, dx + C$$

Example:

$$dy/dx + y = x$$

4. BERNOULLI'S DIFFERENTIAL EQUATION

Standard form:

$$dy/dx + Py = Qy^n$$

Where $n \neq 0$ and $n \neq 1$

Solution Method:

Substitute:

$$z = y^{1-n}$$

Then equation becomes linear differential equation.

5. EXACT DIFFERENTIAL EQUATION

Standard form:

$$Mdx + Ndy = 0$$

Condition for exactness:

$$\partial M/\partial y = \partial N/\partial x$$

Solution Steps:

1. Check exactness condition
2. Integrate M with respect to x
3. Integrate remaining terms with respect to y

6. DIFFERENTIAL EQUATIONS OF FIRST ORDER AND HIGHER DEGREE

Equations having first derivative but degree greater than one.

Methods of Solution:

- Solvable for p
- Solvable for y
- Solvable for x
- Clairaut's equation

7. CLAIRAUT'S EQUATION

Standard form:

$$y = px + f(p)$$

Where $p = dy/dx$

General Solution:

$$y = cx + f(c)$$

Singular Solution:

Obtained by differentiating general solution.

8. HIGHER ORDER DIFFERENTIAL EQUATIONS

Equations containing derivatives of order greater than one.

General Form:

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 \frac{dy}{dx} + a_0 y = X$$

9. LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

General form:

$$f(D)y = X$$

Where $D = d/dx$

Solution:

General solution = Complementary Function + Particular Integral

(a) Complementary Function (C.F.)

Obtained by solving auxiliary equation.

(b) Particular Integral (P.I.)

Depends upon right hand side function X.

10. AUXILIARY EQUATION

Replace D by m in differential equation.

Example:

$$(D^2 + 3D + 2)y = 0$$

Auxiliary equation:

$$m^2 + 3m + 2 = 0$$

Cases of Roots:

- Real and distinct roots
- Equal roots
- Complex roots

11. HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION

Equation with right hand side equal to zero.

Example:

$$d^2y/dx^2 + 4dy/dx + 3y = 0$$

12. SIMULTANEOUS DIFFERENTIAL EQUATIONS

Two or more differential equations solved together.

Example:

$$dx/dt + 2x + y = 0$$

$$dy/dt + x + 2y = 0$$

Solution Method:

- Elimination method
- Operator method

13. APPLICATIONS OF DIFFERENTIAL EQUATIONS

- Electrical circuits
- Population growth
- Mechanical vibrations
- Heat transfer
- Fluid flow

MOST IMPORTANT 14 MARK QUESTIONS

1. Solve linear differential equations using integrating factor method.
2. Solve Bernoulli's differential equation.
3. Solve exact differential equations.
4. Solve differential equations of first order and higher degree.
5. Solve Clairaut's differential equation.
6. Solve higher order differential equations with constant coefficients.
7. Explain complementary function and particular integral.
8. Solve homogeneous linear differential equations.
9. Solve simultaneous differential equations using elimination method.
10. Solve differential equations with repeated and complex roots.

IMPORTANT 7 MARK QUESTIONS

1. Define differential equation, order and degree.
2. Explain integrating factor method.
3. Explain Bernoulli's equation.
4. Explain exact differential equation.
5. Explain auxiliary equation.
6. Explain complementary function.
7. Explain particular integral.
8. Explain simultaneous differential equations.

EXAM TIPS

- Practice numerical problems daily.
- Learn formulas of C.F. and P.I. carefully.
- Revise integrating factor method regularly.
- Practice solving auxiliary equations.
- Focus on repeated university numerical problems.