Mathematics-I Unit-5 Detailed Notes
Matrices

1. Rank of a Matrix

Definition:

Rank of a matrix is the maximum number of linearly independent rows or columns in a matrix.

In simple words, rank tells: How many rows are independent How much information matrix contains **Methods to Find Rank:** Elementary row transformation Determinant method **Example:**

A = [1 2]
[2 4]

Second row is multiple of first row.

Therefore:
Rank(A)=1

2. Elementary Transformations

Elementary transformations are operations performed on rows or columns of matrix.

Types of Elementary Operations: Interchange of rows Multiply row by non-zero number Add multiple of one row to another **Purpose:** Find rank Solve equations Convert matrix into echelon form **Example:**

$R2 \rightarrow R2 - 2R1$ This means: subtract 2 times row1 from row2.

3. Solution of Simultaneous Linear Equations

Simultaneous linear equations can be solved using elementary transformations.

Example:

$x + y = 5$

$x – y = 1$

Add equations:

$2x = 6$

$x = 3$

Substitute in first equation:

$3 + y = 5$

$y = 2$

Therefore:

$x = 3, y = 2$

4. Consistency of Equations

A system of equations is called consistent if it has at least one solution.

Types: Consistent system Inconsistent system **Conditions:**

If: Rank(A)=Rank([A:B])

Then system is consistent.

If: Rank(A) ≠ Rank([A:B])

Then system is inconsistent.

5. Eigen Values and Eigen Vectors

Definition:

For matrix A, if: $Av = \lambda v$

then: λ is Eigen Value v is Eigen Vector **Characteristic Equation:**

$|A - \lambda I| = 0$

Example:

A = [2 0]
[0 3]

Eigen values are: 2 and 3

6. Diagonalization of Matrix

A matrix is diagonalizable if it can be converted into diagonal matrix.

Formula:

$A = PDP^{-1}$

where: D = diagonal matrix P = eigen vector matrix **Steps:** Find eigen values Find eigen vectors Form matrix P Find diagonal matrix D **Applications:** Engineering calculations Machine Learning Computer Graphics

7. Cayley-Hamilton Theorem

Statement:

Every square matrix satisfies its own characteristic equation.

If characteristic equation is:

$\lambda^2 - 5\lambda + 6 = 0$

then matrix satisfies:

$A^2 - 5A + 6I = 0$

8. Application of Cayley-Hamilton Theorem

Cayley-Hamilton theorem is used to: Find inverse of matrix Find powers of matrix Simplify matrix calculations **Formula for Inverse:**

$A^{-1} = 1/\det(A) \times \text{adj}(A)$

Using Cayley-Hamilton theorem, inverse can be calculated easily for square matrices.

Most Important PYQs

Repeated Questions: Find rank of matrix Solve simultaneous equations Check consistency of equations Find eigen values and eigen vectors Diagonalize matrix State and prove Cayley-Hamilton theorem Find inverse using Cayley-Hamilton theorem Most Important 7 Marks Questions: Find eigen values and eigen vectors Explain diagonalization State Cayley-Hamilton theorem Find inverse using Cayley-Hamilton theorem