Mathematics-I Unit-4 Detailed Notes
Vector Spaces

1. Vector Space

A vector space is a collection of vectors where vector addition and scalar multiplication are possible. Example: $v1 = (1,2)$ $v2 = (3,4)$ Addition: $v1 + v2 = (4,6)$ Scalar multiplication: $2v1 = (2,4)$ Properties of vector space: 1. Closure under addition 2. Closure under scalar multiplication 3. Associative property 4. Commutative property 5. Existence of zero vector Applications: Machine Learning, AI, Graphics and Engineering Mathematics.

2. Vector Subspace

A subset of vector space which itself satisfies all vector space properties is called subspace. Conditions: 1. Zero vector must belong to subspace 2. Closed under addition 3. Closed under scalar multiplication Example: $S = \{(x,y,0)\}$ is a subspace of R3. Because: $(x1,y1,0)+(x2,y2,0) =(x1+x2,y1+y2,0)$ and $k(x,y,0)=(kx,ky,0)$

3. Linear Combination

When vectors are multiplied by scalars and added together, the result is called linear combination. Formula: $a1v1 + a2v2 + a3v3$ Example: $v1=(1,2)$ $v2=(3,4)$ Find: $2v1 + 3v2$ Solution: $2(1,2)+3(3,4) =(2,4)+(9,12) =(11,16)$

4. Linearly Dependent Vectors

Vectors are linearly dependent if one vector can be written as combination of other vectors. Condition: $a1v1 + a2v2 + ... + anvn = 0$ where coefficients are not all zero. Example: $v1=(1,2)$ $v2=(2,4)$ Since: $v2 = 2v1$ Therefore vectors are linearly dependent.

5. Linearly Independent Vectors

Vectors are linearly independent if no vector can be written using other vectors. Condition: $a_1v_1 + a_2v_2 + ... + a_nv_n = 0$ implies $a_1 = a_2 = ... = a_n = 0$ Example: $v_1=(1,0)$ $v_2=(0,1)$ These vectors are linearly independent.

6. Basis of Vector Space

A set of linearly independent vectors that span complete vector space is called basis. Example: For R^2: (1,0) and (0,1) form basis vectors. Important Points: 1. Basis vectors must be independent 2. Basis vectors must span complete vector space Dimension means number of basis vectors.

7. Linear Transformation

A transformation T:V to W is called linear transformation if: 1. T(u+v)=T(u)+T(v) 2. T(cv)=cT(v) Example: T(x,y)=(2x,2y) Applications: 1. Computer Graphics 2. AI 3. Image Processing 4. Engineering Mathematics

Most Important PYQs

Repeated Questions: 1. Define vector space with properties 2. Explain vector subspace 3. Differentiate dependent and independent vectors 4. Define basis and dimension 5. Explain linear transformation 6. Find linear combination of vectors