

Mathematics-I Unit-2 Detailed Notes

RGPV Engineering

1. Definite Integral as a Limit of Sum

Definition:

Definite integral ka use area under curve find karne ke liye hota hai. Area ko small rectangles me divide kiya jata hai.

Formula:

Integral from a to b of $f(x)dx = \text{limit of summation } f(x_i)\Delta x$

Example:

Evaluate Integral from 0 to 1 of x^2dx

Solution:

Integral $x^2dx = x^3/3$

Applying limits:

$[x^3/3]$ from 0 to 1

$=1/3$

2. Beta Function

Definition:

Beta function is defined as:

$B(m,n) = \text{Integral from 0 to 1 } x^{m-1}(1-x)^{n-1}dx$

Properties:

1. $B(m,n) = B(n,m)$
2. $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Example:

Find $B(1,1)$

Solution:

$B(1,1) = \text{Integral from 0 to 1 } dx$

$= [x]$ from 0 to 1

$= 1$

3. Gamma Function

Definition:

Gamma function is:

$\Gamma(n) = \int_0^{\infty} e^{-x} x^{(n-1)} dx$

Property:

$\Gamma(n+1) = n\Gamma(n)$

Relation with Factorial:

$\Gamma(n+1) = n!$

Example:

Find $\Gamma(4)$

Solution:

$\Gamma(4) = 3!$

$= 6$

4. Surface Area of Revolution

Definition:

When a curve rotates around an axis, surface generated is called surface of revolution.

Formula about x-axis:

$S = 2\pi \int y \sqrt{1+(dy/dx)^2} dx$

Example:

Find surface area generated by rotating $y=x$ from 0 to 1 about x-axis.

Solution:

$dy/dx=1$

$S=2\pi \int_0^1 x\sqrt{2} dx$

$$= \pi\sqrt{2}$$

5. Volume of Revolution

Formula:

$$V = \pi \int y^2 dx$$

Example:

Find volume generated by rotating $y=x$ from 0 to 1 about x-axis.

Solution:

$$V = \pi \int_0^1 x^2 dx$$

$$= \pi \left[\frac{x^3}{3} \right]_0^1$$

$$= \pi/3$$

6. Multiple Integrals

Definition:

Integration involving more than one variable is called multiple integration.

Double Integral and Triple Integral are examples.

Applications:

Used to find: Area Volume Mass Density **Example:**

Evaluate double integral of $(x+y)$

Answer = 1

7. Change the Order of Integration

Meaning:

Sometimes integration difficult hota hai. Then we reverse the order of integration.

Steps:

1. Draw graph
2. Identify limits
3. Rewrite limits
4. Reverse order

Example:

Integral from 0 to 1 Integral from x to 1 $f(x,y)dydx$

After changing order:

Integral from 0 to 1 Integral from 0 to y $f(x,y)dx dy$

8. Applications of Multiple Integrals

Area Using Double Integral:

$A = \text{Double Integral } dA$

Volume Using Triple Integral:

$V = \text{Triple Integral } dV$

Example:

Find area bounded by: $x=0$, $y=0$ and $x+y=1$

Solution:

Area = $1/2$