Mathematics 1 – Module 1 Easy Notes (RGPV)

Calculus Notes

Topics Covered

- Rolle's Theorem
- Mean Value Theorems
- Maclaurin Series
- Taylor Series (One Variable)
- Taylor Series (Two Variables)
- Partial Differentiation
- Maxima & Minima
- Lagrange Multipliers

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1. Rolle's Theorem

Statement

If a function $f(x)$:

1. is continuous on $[a, b]$
2. differentiable on (a, b)
3. and $f(a) = f(b)$

then there exists at least one point $c \in (a, b)$ such that:

$$f'(c) = 0$$

Geometrical Meaning

If starting point and ending point are at same height, then at some point tangent becomes horizontal.

Steps to Solve Problems

Step 1

Check continuity.

Step 2

Check differentiability.

Step 3

Check:

$$f(a) = f(b)$$

Step 4

Differentiate:

$$f'(x)$$

Step 5

Put:

$$f'(x) = 0$$

Find value of c.

Example

Show Rolle's theorem for:

$$f(x) = x^2 - 4x + 3$$

on interval:

$$[1, 3]$$

Solution

$$f(1) = 1 - 4 + 3 = 0$$

$$f(3) = 9 - 12 + 3 = 0$$

So:

$$f(1) = f(3)$$

Differentiate:

$$f'(x) = 2x - 4$$

Put:

$$2x - 4 = 0$$

$$x = 2$$

Hence:

$$c = 2$$

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2. Mean Value Theorems

(A) Lagrange Mean Value Theorem (LMVT)

Statement

If function is:

- continuous on $[a, b]$
- differentiable on (a, b)

then there exists:

$$c \in (a, b)$$

such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Formula

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Steps

1. Check continuity.
2. Check differentiability.
3. Find:

$$\frac{f(b) - f(a)}{b - a}$$

1. Find derivative.
2. Equate both.

(B) Cauchy Mean Value Theorem

Formula

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

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3. Maclaurin Series

Formula

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$$

Important Expansions

1.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

2.

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

3.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

4.

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

5.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

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4. Taylor Series (One Variable)

Formula

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots$$

Difference Between Taylor & Maclaurin

Taylor Series

Expansion about:

$$x = a$$

Maclaurin Series

Expansion about:

$$x = 0$$

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5. Taylor Series for Function of Two Variables

Suppose:

$$z = f(x, y)$$

Then expansion becomes:

$$f(a+h, b+k)$$

using partial derivatives.

Formula (Short Form)

$$f(a+h, b+k) = f(a, b)$$

$$+hf_x + kf_y$$

$$+\frac{1}{2!}(h^2 f_{xx} + 2hkf_{xy} + k^2 f_{yy}) + \cdots$$

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6. Partial Differentiation

Definition

If function depends on multiple variables:

$$z = f(x, y)$$

then derivative with respect to one variable while keeping other constant is called partial derivative.

Symbols

Partial derivative wrt x

$$\frac{\partial z}{\partial x}$$

Partial derivative wrt y

$$\frac{\partial z}{\partial y}$$

Example

$$z = x^2 y + 3xy^2$$

wrt x

$$\frac{\partial z}{\partial x} = 2xy + 3y^2$$

wrt y

$$\frac{\partial z}{\partial y} = x^2 + 6xy$$

Higher Order Partial Derivatives

Second Order

$$\frac{\partial^2 z}{\partial x^2}$$

$$\frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial^2 z}{\partial x \partial y}$$

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7. Maxima and Minima

Conditions

For:

$$z = f(x, y)$$

Step 1

Find:

$$\frac{\partial z}{\partial x} = 0$$

and

$$\frac{\partial z}{\partial y} = 0$$

Step 2

Find:

$$r = f_{xx}$$

$$s = f_{xy}$$

$$t = f_{yy}$$

Step 3

Compute:

$$D = rt - s^2$$

Result

If:

$$D > 0$$

and

$$r > 0$$

then Minimum.

If:

$$D > 0$$

and

$$r < 0$$

then Maximum.

If:

$$D < 0$$

then Saddle Point.

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8. Lagrange Multipliers

Used for maxima/minima with constraints.

Constraint Form

$$g(x, y) = 0$$

Formula

$$\nabla f = \lambda \nabla g$$

or:

$$f_x = \lambda g_x$$

$$f_y = \lambda g_y$$

Steps

Step 1

Write:

$$f(x, y)$$

and constraint:

$$g(x, y) = 0$$

Step 2

Find partial derivatives.

Step 3

Use:

$$f_x = \lambda g_x$$

$$f_y = \lambda g_y$$

Step 4

Solve equations.

Important Exam Questions

Very Important 7 Marks Questions

1. State and prove Rolle's theorem.
2. State and prove Lagrange Mean Value theorem.
3. Expand e^x using Maclaurin series.
4. Expand $\sin x$ using Maclaurin series.
5. Explain partial differentiation with examples.
6. Find maxima and minima of functions of two variables.
7. Explain method of Lagrange multipliers.
8. Expand function using Taylor series.

Quick Revision Formula Sheet

Rolle's Theorem

$$f'(c) = 0$$

LMVT

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Maclaurin Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$$

Taylor Series

$$f(x) = f(a) + (x - a)f'(a) + \cdots$$

Maxima-Minima Test

$$D = rt - s^2$$

Last Minute Tips

- Learn all standard expansions.
- Practice derivatives carefully.
- Remember maxima-minima conditions.
- Learn formulas of Taylor & Maclaurin.
- Solve previous year numerical questions.
- Practice partial differentiation daily.

End of Notes