

# Posets, Hasse Diagram and Lattices

## 1. Introduction to Posets

A **Poset** means:

Partially Ordered Set

It is a set together with a relation that satisfies:

1. Reflexive
2. Antisymmetric
3. Transitive

A Poset is represented as:

$(P, \leq)$

Where:

- $P$  = set
  - $\leq$  = partial order relation
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## 2. Ordered Set

A set in which elements are arranged according to a relation is called an ordered set.

Example:

$A = \{1, 2, 3, 4\}$

Relation:

$\leq$

Then:

$(A, \leq)$

is an ordered set.

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### 3. Partial Ordered Relation

A relation  $R$  on set  $A$  is called partially ordered if:

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#### (i) Reflexive

Every element is related to itself.

$aRa$

Example:

$1 \leq 1$

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#### (ii) Antisymmetric

If:

$aRb$  and  $bRa$  then  $a=b$

then:

$a=b$

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#### (iii) Transitive

If:

$aRb$  and  $bRc$  then  $aRc$

then:

## 4. Hasse Diagram

A graphical representation of partially ordered sets is called Hasse Diagram.

It removes:

- Reflexive edges
  - Transitive edges
  - Arrow directions
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### Rules for Drawing Hasse Diagram

1. Greater elements are placed upward.
  2. Smaller elements are placed downward.
  3. No arrows are used.
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### Example

Set:

$$A = \{1, 2, 4, 8\}$$

Relation:

“Divides”

Hasse Diagram:



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## 5. Isomorphic Ordered Set

Two ordered sets are isomorphic if they have same structure.

There exists one-to-one correspondence preserving order relation.

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### Conditions

1. Same number of elements
2. Same ordering structure

Applications:

- Computer science
  - Data organization
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## 6. Well Ordered Set

A partially ordered set is well ordered if:

Every non-empty subset has a least element.

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### Example

Natural numbers:

$N = \{1, 2, 3, 4, \dots\}$

Every subset has smallest element.

Hence it is well ordered.

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## 7. Lattices

A lattice is a poset in which every pair of elements has:

1. Greatest Lower Bound (GLB)
  2. Least Upper Bound (LUB)
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### GLB (Meet)

Represented by:

$$a \wedge b \text{ \wedge } b \wedge a$$

Largest element smaller than both.

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### LUB (Join)

Represented by:

$$a \vee b \text{ \vee } b \vee a$$

Smallest element greater than both.

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## 8. Properties of Lattices

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### (i) Idempotent Law

$$a \vee a = a \text{ \vee } a = a \vee a \quad a \wedge a = a \text{ \wedge } a = a \wedge a$$

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## (ii) Commutative Law

$$a \vee b = b \vee a \quad a \wedge b = b \wedge a$$

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## (iii) Associative Law

$$(a \vee b) \vee c = a \vee (b \vee c) \quad (a \wedge b) \wedge c = a \wedge (b \wedge c)$$

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## (iv) Absorption Law

$$a \vee (a \wedge b) = a \quad a \wedge (a \vee b) = a$$

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# 9. Bounded Lattice

A lattice having:

- Greatest element (1)
- Least element (0)

is called bounded lattice.

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# 10. Complemented Lattice

A bounded lattice where every element has complement.

If:

$$a \vee a' = 1 \quad a \wedge a' = 0$$

and

$$a \wedge a' = 0 \quad a \vee a' = 1$$

then:

a'a'a'

is complement of aaa.

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# COMBINATORICS

## 11. Introduction to Combinatorics

Combinatorics deals with:

- Counting
- Arrangement
- Selection

Applications:

- Probability
  - Cryptography
  - Computer science
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## 12. Permutation

Arrangement of objects in specific order.

Formula:

$${}^n P_r = \frac{n!}{(n-r)!} \quad {}^n P_r = (n-r)!n!$$

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### Example

Arrange 3 letters from 5 letters:

$$5P3 = 5!2! = 60 \quad 5P_3 = \frac{5!}{2!} = 60 \quad 5P3 = 2!5! = 60$$

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## 13. Combination

Selection of objects without order.

Formula:

$$nC_r = \frac{n!}{r!(n-r)!} \quad nC_r = \frac{n!}{r!(n-r)!} \quad nC_r = r!(n-r)!n!$$

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### Example

Choose 2 students from 5:

$$5C_2 = 10 \quad 5C_2 = 10 \quad 5C_2 = 10$$

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## 14. Difference Between Permutation and Combination

Permutation	Combination
Order matters	Order does not matter
Arrangement	Selection

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## 15. Binomial Theorem

Used to expand:

$$(a+b)^n$$

Formula:

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$


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## Example

$$(a+b)^2 = a^2 + 2ab + b^2$$

Applications:

- Algebra
  - Probability
  - Computer algorithms
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# 16. Multinomial Coefficients

Extension of binomial theorem.

Formula:

$$(x_1 + x_2 + \dots + x_k)^n$$

Coefficient:

$$\frac{n!}{n_1! n_2! \dots n_k!}$$

Where:

$$n_1 + n_2 + \dots + n_k = n$$


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# RECURRENCE RELATION

## 17. Introduction to Recurrence Relation

An equation defining sequence using previous terms.

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## Example

$$a_n = a_{n-1} + 2a_{n-2}$$

Applications:

- Algorithms
  - Fibonacci series
  - Dynamic programming
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## 18. Recursive Algorithms

Algorithms calling themselves repeatedly.

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### Example: Factorial

$$n! = n(n-1)!(n-1)! = n(n-1)!$$

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## 19. Linear Recurrence Relation

General form:

$$a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots = 0$$

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## 20. Linear Recurrence with Constant Coefficients

Coefficients remain constant.

Example:

$$a_n - 5a_{n-1} + 6a_{n-2} = 0 \quad a_n - 5a_{n-1} + 6a_{n-2} = 0$$

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## 21. Homogeneous Solution

If RHS = 0

Example:

$$a_n - 3a_{n-1} = 0 \quad a_n - 3a_{n-1} = 0$$

Characteristic equation:

$$r - 3 = 0 \quad r - 3 = 0 \quad r = 3 \quad r = 3$$

Solution:

$$a_n = C(3^n) \quad a_n = C(3^n)$$

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## 22. Particular Solution

Used when RHS  $\neq 0$

Example:

$$a_n - a_{n-1} = 2 \quad a_n - a_{n-1} = 2$$

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## 23. Total Solution

Total Solution = Homogeneous Solution + Particular Solution  
 $\text{\text{Total Solution}} = \text{\text{Homogeneous Solution}} + \text{\text{Particular}}$

Solution} Total Solution = Homogeneous Solution + Particular Solution

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## 24. Generating Function

A power series representing sequence.

Formula:

$$G(x) = a_0 + a_1x + a_2x^2 + \dots \quad G(x) = a_0 + a_1x + a_2x^2 + \dots$$

Applications:

- Solving recurrence relations
  - Counting problems
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## 25. Solving by Generating Function

Steps:

1. Form generating function
  2. Manipulate algebraically
  3. Expand series
  4. Find sequence terms
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## Important Exam Questions

### 7-Mark Questions

1. Define Poset with suitable example.
2. Explain Hasse Diagram.
3. Define lattice and its properties.

4. Differentiate permutation and combination.
  5. Explain recurrence relation with example.
  6. Define generating function.
  7. Explain complemented lattice.
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## 14-Mark Questions

1. Explain Posets and Hasse Diagram in detail.
  2. Discuss properties of lattices with examples.
  3. Explain permutation and combination with formulas.
  4. Explain Binomial theorem with examples.
  5. Explain recurrence relation and generating functions.
  6. Solve recurrence relation using generating function method.
  7. Explain bounded and complemented lattices.
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## Quick Revision Formula Sheet

### Permutation

$${}^n P_r = \frac{n!}{(n-r)!} \quad {}^n P_n = n!$$


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### Combination

$${}^n C_r = \frac{n!}{r!(n-r)!} \quad {}^n C_n = 1$$


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### Binomial Theorem

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$


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# Generating Function

$$G(x) = a_0 + a_1x + a_2x^2 + \dots \quad G(x) = a_0 + a_1x + a_2x^2 + \dots$$

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## Applications

Topic	Applications
Posets	Scheduling
Hasse Diagram	Hierarchical structures
Lattices	Boolean algebra
Permutation	Arrangements
Combination	Selection problems
Recurrence Relation	Algorithms
Generating Function	Sequence solving

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## Conclusion

These topics are very important in:

- Discrete Mathematics
- Data Structures
- Algorithms
- Compiler Design
- Artificial Intelligence
- Competitive Programming