7-Mark Questions with Detailed Easy Explanation

1. Define Poset with Suitable Example

Definition

A **Poset** means:

> Partially Ordered Set

A set together with a relation is called Poset if the relation satisfies:

1. Reflexive
2. Antisymmetric
3. Transitive

It is represented as:

(P,R)(P,R)(P,R)

Where:

- PPP = set
- RRR = relation

Conditions of Poset

1. Reflexive

Every element is related to itself.

aRaaRaaRa

Example:

1≤11 \leq 11≤1

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2. Antisymmetric

If:

aRb and bRaaRb \text{ and } bRaaRb and bRa

then:

a=ba=ba=b

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3. Transitive

If:

aRb and bRcaRb \text{ and } bRcaRb and bRc

then:

aRcaRcaRc

Example

Set:

A={1,2,4,8}A=\{1,2,4,8\}A={1,2,4,8}

Relation:

“Divides”

1 ∣2, 2 ∣4, 4 ∣81|2,\ 2|4,\ 4|81 ∣2, 2 ∣4, 4 ∣8

This forms a Poset.

Applications

- Scheduling
- Database systems
- Hierarchical structures

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2. Explain Hasse Diagram

Definition

A **Hasse Diagram** is a graphical representation of partially ordered sets.

It simplifies the diagram by removing:

- Reflexive edges
- Transitive edges
- Arrow directions

Rules for Drawing Hasse Diagram

1. Larger elements placed upward.
2. Smaller elements placed downward.
3. No arrows used.

Example

Set:

$A=\{1,2,4,8\}$

Relation:

“Divides”

Hasse Diagram:

```
8
|
4
|
2
|
1
```

Advantages

- Easy visualization
- Simplifies relations
- Used in lattice representation

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3. Define Lattice and its Properties

Definition

A lattice is a Poset where every pair of elements has:

1. Greatest Lower Bound (GLB)
2. Least Upper Bound (LUB)

Meet and Join

Meet (GLB)

Represented by:

$a \wedge b$

Largest element smaller than both.

Join (LUB)

Represented by:

$a \vee b$

Smallest element greater than both.

Properties of Lattice

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1. Idempotent Law

$a \vee a = a$ $a \wedge a = a$

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2. Commutative Law

$a \vee b = b \vee a$ $a \wedge b = b \wedge a$

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3. Associative Law

$(a \vee b) \vee c = a \vee (b \vee c)$

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4. Absorption Law

$a \vee (a \wedge b) = a$

Applications

- Boolean algebra
- Switching circuits
- Computer logic design

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4. Differentiate Permutation and Combination

Permutation	**Combination**
Arrangement of objects	Selection of objects
Order matters	Order does not matter
Formula: nPr=n!(n−r)!^nP_r=\frac{n!}{(n-r)!}nPr=(n−r)!n!	Formula: nCr=n!r!(n−r)!^nC_r=\frac{n!}{r!(n-r)!}nCr=r!(n−r)!n!

Example

Permutation

Arrange 2 letters from A,B,C

AB, BA, AC, CA

Order changes.

Combination

Choose 2 letters from A,B,C

AB, AC, BC

Order does not matter.

Applications

- Seating arrangements
- Password generation
- Team selection

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5. Explain Recurrence Relation with Example

Definition

A recurrence relation defines sequence terms using previous terms.

Example

an=an−1+2a_n=a_{n-1}+2an=an−1+2

If:

a1=1a_1=1a1=1

then:

a2=3a_2=3a2=3 a3=5a_3=5a3=5 a4=7a_4=7a4=7

Types

1. Linear recurrence
2. Non-linear recurrence
3. Homogeneous recurrence

Applications

- Fibonacci series
- Algorithm analysis
- Dynamic programming

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6. Define Generating Function

Definition

A generating function is a power series representing sequence terms.

Formula:

G(x)=a0+a1x+a2x2+…G(x)=a_0+a_1x+a_2x^2+\dotsG(x)=a0+a1x+a2x2+…

Example

Sequence:

1,1,1,1,…1,1,1,1,\dots1,1,1,1,…

Generating function:

1+x+x2+x3+…1+x+x^2+x^3+\dots1+x+x2+x3+…

Uses

- Solving recurrence relations
- Counting problems
- Sequence analysis

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7. Explain Complemented Lattice

Definition

A lattice where every element has complement is called complemented lattice.

If:

a∨a′=1a\vee a'=1a∨a′=1

and

a∧a′=0a\wedge a'=0a∧a′=0

then:

a′a'a′

is complement of aaa.

Example

Boolean algebra is complemented lattice.

Applications

- Logic circuits
- Boolean algebra
- Digital systems

14-Mark Questions

1. Explain Posets and Hasse Diagram in Detail

Poset

A partially ordered set satisfying:

- Reflexive
- Antisymmetric
- Transitive

Example

$A=\{1,2,4,8\}$

with divisibility relation.

Hasse Diagram

Used to represent Poset graphically.

Rules:

1. Remove arrows
2. Remove transitive edges
3. Arrange higher elements upward

Advantages

- Simple representation
- Easy understanding
- Used in lattices

Applications

- Scheduling

- Data hierarchy
- Dependency systems

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2. Discuss Properties of Lattices with Examples

Definition

Lattice is a Poset having LUB and GLB.

Properties

1. Idempotent

a∨a=aa\vee a=aa∨a=a

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2. Commutative

a∨b=b∨aa\vee b=b\vee aa∨b=b∨a

---

3. Associative

(a∨b)∨c=a∨(b∨c)(a\vee b)\vee c=a\vee(b\vee c)(a∨b)∨c=a∨(b∨c)

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4. Absorption

a∨(a∧b)=aa\vee(a\wedge b)=aa∨(a∧b)=a

Example

Boolean algebra forms lattice.

Applications

- Computer circuits
- Logic design
- Switching theory

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3. Explain Permutation and Combination with Formulas

Permutation

Arrangement where order matters.

Formula:

nPr=n!(n−r)!^nP_r=\frac{n!}{(n-r)!}nPr=(n−r)!n!

Example

5P2=20^5P_2=205P2=20

Combination

Selection where order does not matter.

Formula:

nCr=n!r!(n−r)!^nC_r=\frac{n!}{r!(n-r)!}nCr=r!(n−r)!n!

Example

5C2=10^5C_2=105C2=10

Applications

- Team formation
- Password systems
- Seating arrangement

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4. Explain Binomial Theorem with Examples

Statement

Used to expand:

(a+b)n(a+b)^n(a+b)n

Formula:

(a+b)n=∑r=0n(nr)an−rbr(a+b)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r(a+b)n=∑r=0n(rn
)an−rbr

Example

(a+b)2=a2+2ab+b2(a+b)^2=a^2+2ab+b^2(a+b)2=a2+2ab+b2

Applications

- Algebra
- Probability
- Engineering mathematics

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5. Explain Recurrence Relation and Generating Functions

Recurrence Relation

Defines terms using previous terms.

Example:

$$a_n=a_{n-1}+2$$

Generating Function

Represents sequence as power series.

$$G(x)=a_0+a_1x+a_2x^2+\dots$$

Applications

- Algorithm design
- Sequence solving
- Counting problems

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6. Solve Recurrence Relation Using Generating Function Method

Example

an=an−1+1a_n=a_{n-1}+1an=an−1+1

with:

a0=0a_0=0a0=0

Step 1

Sequence:

0,1,2,3,4,…0,1,2,3,4,\dots0,1,2,3,4,…

Step 2

Generating function:

G(x)=0+x+2x2+3x3+…G(x)=0+x+2x^2+3x^3+\dotsG(x)=0+x+2x2+3x3+…

Step 3

Simplify:

G(x)=x(1−x)2G(x)=\frac{x}{(1-x)^2}G(x)=(1−x)2x

Final Solution

an=na_n=nan=n

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7. Explain Bounded and Complemented Lattices

Bounded Lattice

A lattice having:

- Greatest element (1)
- Least element (0)

Example

Boolean algebra.

Complemented Lattice

Every element has complement.

Conditions:

$a \vee a' = 1$ $a \wedge a' = 0$

Applications

- Boolean algebra
- Logic circuits
- Digital electronics

Important Tips for Exams

Most Repeated Topics

1. Hasse Diagram
2. Lattice properties
3. Permutation and Combination
4. Binomial theorem
5. Recurrence relation
6. Generating function

Quick Formula Revision

Permutation

$$^nP_r=\frac{n!}{(n-r)!}$$

Combination

$$^nC_r=\frac{n!}{r!(n-r)!}$$

Binomial Theorem

$$(a+b)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$$