

# Discrete Structure (CSIT-302)

## UNIT-3 Important Questions with Detailed Solutions

### Propositional Logic & Finite State Machines

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## ★ MOST IMPORTANT QUESTIONS FOR RGPV EXAM

Topic	Importance
Truth Tables	★★★★★
Tautology & Contradiction	★★★★★
Logical Equivalence	★★★★★
DeMorgan's Laws	★★★★★
Predicate Logic	★★★★★
Quantifiers	★★★★★
CNF & DNF	★★★★★
Finite State Machine	★★★★★
FSM as Language Recognizer	★★★★★

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### Q1. Construct Truth Table for

$$(p \wedge q) \rightarrow p \text{ to } p(p \wedge q) \rightarrow p$$

and determine whether it is tautology.

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## Solution

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### Step 1: Create Columns

We need columns for:

- $p$
  - $q$
  - $p \wedge q$
  - $(p \wedge q) \rightarrow p$
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### Truth Table

$p$	$q$	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

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### Final Observation

Last column contains all TRUE.

Hence:

$$(p \wedge q) \rightarrow p \text{ is a tautology}$$

is a **tautology**.

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## Q2. Explain Tautology and Contradiction with Examples.

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### Solution

### Tautology

A proposition always true for every truth value.

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### Example

$$p \vee \neg p$$

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### Truth Table

$p$	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Hence tautology.

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### Contradiction

A proposition always false.

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## Example

$$p \wedge \neg p \text{ and } \neg p \wedge p$$

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## Truth Table

$p$	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Hence contradiction.

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## Difference

Tautology	Contradiction
Always true	Always false
Example: $p \vee \neg p$ or $\neg p \vee p$	Example: $p \wedge \neg p$ and $\neg p \wedge p$

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## Q3. Prove Using Logical Equivalence

$$p \rightarrow q \equiv \neg p \vee q \text{ to } q \equiv \neg p \vee q$$

$$qp \rightarrow q \equiv \neg p \vee q$$

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# Solution

We construct truth table.

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## Truth Table

<b>p</b>	<b>q</b>	<b><math>p \rightarrow q</math></b>	<b><math>\neg p</math></b>	<b><math>\neg p \vee q</math></b>
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

## Observation

Columns:

$$p \rightarrow q \text{ to } q \rightarrow q$$

and

$$\neg p \vee q \text{ neg } p \text{ lor } q \neg p \vee q$$

are same.

Hence:

$$p \rightarrow q \equiv \neg p \vee q \text{ to } q \equiv \text{equiv } \text{neg } p \text{ lor}$$

$$q \rightarrow q \equiv \neg p \vee q$$

proved.

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## Q4. Explain DeMorgan's Laws.

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### Solution

#### First Law

$$\neg(p \wedge q) = \neg p \vee \neg q \quad \text{neg}(p \text{ \& } q) = \text{neg } p \text{ \lor } \text{neg } q$$
$$q \neg(p \wedge q) = \neg p \vee \neg q$$

Meaning:

Negation of AND becomes OR of negations.

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#### Second Law

$$\neg(p \vee q) = \neg p \wedge \neg q \quad \text{neg}(p \text{ \lor } q) = \text{neg } p \text{ \& } \text{neg } q$$
$$q \neg(p \vee q) = \neg p \wedge \neg q$$

Meaning:

Negation of OR becomes AND of negations.

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### Verification Using Truth Table

## First Law

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Both columns same.

Hence proved.

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## Q5. Explain Predicate Logic with Example.

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### Solution

### Predicate

A statement depending on variable.

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### Example

$P(x): x > 5$   $P(x): x > 5$   $P(x): x > 5$

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### Domain

Suppose:

$x \in \mathbb{N} \mid x \in \mathbb{N}$

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## Verification

If:

$$x = 7$$

then:

$$P(7) = \text{TRUE}$$

If:

$$x = 3$$

then:

$$P(3) = \text{FALSE}$$

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## Importance

Predicate logic helps represent mathematical statements clearly.

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## Q6. Explain Universal and Existential Quantifiers.

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### Solution

#### Universal Quantifier

Symbol:

$\forall$  for all  $\forall$

Meaning:

“For all”

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#### Example

$\forall x(x+0=x)$  for all  $x(x+0=x)$   $\forall x(x+0=x)$

means every number added with 0 remains same.

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#### Existential Quantifier

Symbol:

$\exists$  exists  $\exists$

Meaning:

“There exists”

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## Example

$$\exists x(x^2=4) \setminus \text{exists } x(x^2=4) \exists x(x^2=4)$$

means there exists number whose square is 4.

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## Difference

Universal	Existential
For all	There exists
Symbol: $\forall$	Symbol: $\exists$

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## Q7. Convert into CNF and DNF

$$(p \wedge q) \vee r(p \setminus \text{land } q) \setminus \text{lor } r(p \wedge q) \vee r$$

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## Solution

### DNF

Expression already in OR of AND form.

Hence DNF:

$$(p \wedge q) \vee r(p \wedge q) \vee r$$

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**CNF**

Use distributive law:

$$\begin{aligned}(p \wedge q) \vee r(p \wedge q) \vee r &= \\(p \vee r) \wedge (q \vee r) &= (p \vee r) \wedge (q \vee r) = \\(p \vee r) \wedge (q \vee r) &\end{aligned}$$

Hence CNF:

$$\begin{aligned}(p \vee r) \wedge (q \vee r) &= (p \vee r) \wedge (q \vee r) \\(p \vee r) \wedge (q \vee r) &\end{aligned}$$

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**Q8. Define Finite State Machine (FSM).**

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**Solution**

**Definition**

Finite State Machine is mathematical model having finite number of states.

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# Components

1. States
  2. Inputs
  3. Outputs
  4. Transition function
  5. Initial state
  6. Final state
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# Example

Traffic Light System

States:

- Red
- Yellow
- Green

Transitions:

Red → Green → Yellow → Red

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# Applications

1. ATM
  2. Compiler design
  3. Digital circuits
  4. Elevator systems
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**Q9. Explain FSM as Language Recognizer.**

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# Solution

FSM can recognize strings/languages.

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# Example

Language of binary strings ending with 1.

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# States

- q0
  - q1
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# Working

If final digit is 1,

machine accepts string.

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# Example Strings

Accepted:

- 1
- 101
- 111

Rejected:

- 100
  - 110
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## **Final Conclusion**

FSM checks whether string belongs to language.

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## **Q10. Explain Equivalence Machines.**

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### **Solution**

### **Definition**

Two FSMs are equivalent if they produce same output for same input sequence.

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### **Meaning**

Machines may look different,

but behavior/output same.

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### **Example**

Two vending machines accepting same coins and giving same product.

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### **Importance**

Used in:

- circuit simplification
- automata optimization
