UNIT–1 : Set Theory, Relation, Function & Theorem Proving Techniques

Discrete Structure (CSIT-302) Notes

Easy Language + RGPV Exam Oriented Notes

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1. SET THEORY

What is a Set?

A **set** is a collection of well-defined objects.

These objects are called **elements** or **members**.

Examples

A={1,2,3,4}A=\{1,2,3,4\}A={1,2,3,4} B={a,e,i,o,u}B=\{a,e,i,o,u\}B={a,e,i,o,u}

Important Points

- Elements are written inside curly brackets {}\{\}{}
- Duplicate elements are not allowed
- Order does not matter

Example:

{1,2,3}={3,2,1}\{1,2,3\}=\{3,2,1\}{1,2,3}={3,2,1}

Types of Sets

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1. Empty Set (Null Set)

A set having no element.

Symbol:

ϕ or {}\phi \text{ or } \{\}ϕ or {}

Example:

Set of students having age 200 years.

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2. Finite Set

A set containing limited elements.

Example:

{1,2,3,4}\{1,2,3,4\}{1,2,3,4}

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3. Infinite Set

A set having unlimited elements.

Example:

N={1,2,3,… }\mathbb{N}=\{1,2,3,\dots\}N={1,2,3,…}

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4. Equal Sets

Two sets are equal if they contain same elements.

Example:

A={1,2,3}A=\{1,2,3\}A={1,2,3} B={3,2,1}B=\{3,2,1\}B={3,2,1}

Then:

A=BA=BA=B

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5. Subset

If every element of AAA is also in BBB, then AAA is subset of BBB.

Symbol:

A⊆BA\subseteq BA⊆B

Example:

{1,2}⊆{1,2,3}\{1,2\}\subseteq\{1,2,3\}{1,2}⊆{1,2,3}

Venn Diagram

A Venn diagram is a pictorial representation of sets.

Used to show:

- Union
- Intersection
- Difference
- Complement

Set Operations

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1. Union

All elements from both sets.

Symbol:

A∪BA\cup BA∪B

Example:

A={1,2}A=\{1,2\}A={1,2} B={2,3}B=\{2,3\}B={2,3} A∪B={1,2,3}A\cup B=\{1,2,3\}A∪B={1,2,3}

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2. Intersection

Common elements.

Symbol:

A∩BA\cap BA∩B

Example:

A∩B={2}A\cap B=\{2\}A∩B={2}

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3. Difference

Elements in AAA but not in BBB.

A−BA-BA−B

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4. Complement

Elements not in set AAA.

Symbol:

A′A'A′

Important Identities on Sets

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1. Commutative Law

A∪B=B∪AA\cup B=B\cup AA∪B=B∪A A∩B=B∩AA\cap B=B\cap AA∩B=B∩A

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2. Associative Law

(A∪B)∪C=A∪(B∪C)(A\cup B)\cup C=A\cup(B\cup C)(A∪B)∪C=A∪(B∪C)

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3. Distributive Law

A∩(B∪C)=(A∩B)∪(A∩C)A\cap(B\cup C)=(A\cap B)\cup(A\cap C)A∩(B∪C)=(A∩B)∪(A∩C)

Countable and Uncountable Sets

Countable Set

A set whose elements can be counted.

Example:

- Natural numbers
- Integers

Uncountable Set

A set whose elements cannot be counted.

Example:

- Real numbers

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2. RELATION

Definition of Relation

A relation shows connection between elements of two sets.

If:

A={1,2}A=\{1,2\}A={1,2} B={a,b}B=\{a,b\}B={a,b}

Then relation:

R={(1,a),(2,b)}R=\{(1,a),(2,b)\}R={(1,a),(2,b)}

Types of Relations

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1. Reflexive Relation

If:

(a,a)∈R(a,a)\in R(a,a)∈R

for every aaa.

Example:

{(1,1),(2,2)}\{(1,1),(2,2)\}{(1,1),(2,2)}

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2. Symmetric Relation

If:

(a,b)∈R(a,b)\in R(a,b)∈R

then:

(b,a)∈R(b,a)\in R(b,a)∈R

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3. Transitive Relation

If:

(a,b)∈R(a,b)\in R(a,b)∈R

and

(b,c)∈R(b,c)\in R(b,c)∈R

then:

(a,c)∈R(a,c)\in R(a,c)∈R

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4. Antisymmetric Relation

If:

$(a,b) \in R$

and

$(b,a) \in R$

then:

$a=b$

Composition of Relations

If:

$R=\{(a,b)\}$

and

$S=\{(b,c)\}$

Then:

$R \circ S=\{(a,c)\}$

Pictorial Representation of Relation

Relations can be represented using:

- Arrow diagrams
- Directed graphs

Equivalence Relation

A relation which is:

1. Reflexive
2. Symmetric
3. Transitive

Example

“Same remainder when divided by 2”

Partial Ordering Relation

A relation which is:

1. Reflexive
2. Antisymmetric
3. Transitive

Example

≤\le≤

relation on integers.

Job Scheduling Problem

Used in partial ordering.

Tasks are arranged according to priority/order.

Example:

- Task A before B
- B before C

Then:

$A \rightarrow B \rightarrow C$

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3. FUNCTION

Definition of Function

A function maps each element of set A to exactly one element of set B.

$f: A \to B$

Example

$f(x) = 2x$

Types of Functions

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1. One-One Function

Different inputs give different outputs.

$f(a) = f(b) \Rightarrow a = b$

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2. Into Function

At least one element in codomain has no preimage.

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3. Onto Function

Every element of codomain has preimage.

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4. Bijective Function

Both one-one and onto.

Inverse Function

If function is bijective then inverse exists.

Symbol:

f^{-1}

Composition of Functions

If:

$f: A \to B$

and

$g: B \to C$

Then:

$g \circ f: A \to C$

Recursively Defined Functions

Function defined using previous values.

Example:

Factorial

n!=n(n−1)!n!=n(n-1)!n!=n(n−1)!

Pigeonhole Principle

Statement

If n+1n+1n+1 objects are placed into nnn boxes,

then at least one box contains two or more objects.

Example

Among 13 people,

at least two have same birth month.

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4. THEOREM PROVING TECHNIQUES

1. Mathematical Induction

Used to prove statements true for all natural numbers.

Steps

Step 1: Base Case

Check for n=1n=1n=1

Step 2: Induction Hypothesis

Assume true for n=kn=kn=k

Step 3: Induction Step

Prove true for k+1k+1k+1

Example

1+2+3+⋯+n=n(n+1)21+2+3+\dots+n=\frac{n(n+1)}{2}1+2+3+⋯+n=2n(n+1)

Proof by Contradiction

Assume opposite statement is true.

If contradiction occurs,

original statement is true.

Example

Prove 2\sqrt22 is irrational.

Assume 2\sqrt22 is rational.

After solving, contradiction occurs.

Hence:

2\sqrt22

is irrational.

MOST IMPORTANT TOPICS FOR RGPV EXAM

⭐ Equivalence Relation
⭐ Mathematical Induction
⭐ Functions
⭐ Pigeonhole Principle
⭐ Partial Ordering Relation
⭐ Set Identities
⭐ Countable & Uncountable Sets
⭐ Proof by Contradiction